

A-Level Further Maths – Core Pure

A-Level Further Maths (Core Pure 1 & 2) · Total marks: 75 · Answers on the last page

Exam-style practice for Core Pure (Edexcel CP1/CP2, AQA and OCR equivalents): complex numbers and de Moivre, matrices, roots of polynomials, proof by induction, hyperbolic functions, Maclaurin series, polar coordinates, further integration and volumes of revolution. A calculator may be used throughout. Allow about 90 minutes.

Section A – Core skills (Give exact answers unless otherwise stated.)

- Given that $1 + 2i$ is a root of $z^3 - 5z^2 + 11z - 15 = 0$, find the other two roots. [4]
- Let $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$.
 - Find A^{-1} .
 - The matrix A represents a transformation T . Describe the effect of T on the area of a shape.
 - Find the eigenvalues and corresponding eigenvectors of $M = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$. [8]
- The roots of $z^3 - 2z^2 + 3z - 1 = 0$ are α , β and γ .
 - Write down $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$.
 - Find $\alpha^2 + \beta^2 + \gamma^2$.
 - Find $\alpha^3 + \beta^3 + \gamma^3$. [6]
- Prove by induction that $7^n + 5$ is divisible by 6 for all positive integers n . [5]

Section B – Exam-style

- Let $z = -2 + 2\sqrt{3}i$.
 - Write z in the form $re^{i\theta}$, $-\pi < \theta \leq \pi$.
 - Hence find z^4 in the form $a + bi$.
 - Find the four roots of $w^4 = -16$, giving your answers in the form $a + bi$. [8]
- Solve $3 \cosh x - 2 \sinh x = 3$, giving answers in exact logarithmic form.
 - Find $\int \frac{1}{\sqrt{x^2 + 4x + 13}} dx$. [7]
- Use standard results to show that $\sum_{r=1}^n (2r-1)^2 = \frac{n(2n-1)(2n+1)}{3}$.
 - Use the method of differences to find $\sum_{r=1}^n \frac{1}{r(r+1)}$ and hence state the sum to infinity. [7]
- Let $B = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 2 & 0 & 1 \end{pmatrix}$.
 - Show that $\det B = 13$.
 - Find B^{-1} .
 - Hence solve the system $x + 2y = 3$, $y + 3z = 4$, $2x + z = 5$. [7]

Section C – Extended

9. (a) Find the Maclaurin series of $\ln(1+2x)$ up to and including the term in x^3 , and state the range of x for which it is valid.
(b) Hence find an approximation for $\ln 1.2$ to 4 decimal places and compare it with the calculator value. [6]
10. The curve C has polar equation $r = 2 + \cos \theta$, $0 \leq \theta < 2\pi$.
(a) Find the area enclosed by C .
(b) Find the exact values of r at the points where the tangent to C is parallel to the initial line, other than where $\theta = \pi$. [Hint: consider $y = r \sin \theta$.] [8]
11. (a) The region bounded by $y = e^{-x}$, the x -axis and the lines $x = 0$ and $x = \ln 2$ is rotated through 2π radians about the x -axis. Find the exact volume of the solid formed.
(b) Find the mean value of $\cosh x$ on the interval $[0, \ln 3]$, giving your answer in exact form and to 3 significant figures.
(c) Evaluate $\int_0^1 \frac{1}{x^2 + 2x + 5} dx$ exactly. [9]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

1. $1 - 2i$ is also a root; $(z - 1 - 2i)(z - 1 + 2i) = z^2 - 2z + 5$, and $z^3 - 5z^2 + 11z - 15 = (z^2 - 2z + 5)(z - 3)$.
Roots: $1 - 2i$ and 3
2. (a) $A^{-1} = \frac{1}{5} \begin{pmatrix} 4 & -1 \\ -3 & 2 \end{pmatrix}$
(b) $\det A = 5$, so areas are multiplied by 5.
(c) $\lambda^2 - 7\lambda + 10 = 0$: $\lambda = 5$, eigenvector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$; $\lambda = 2$, eigenvector $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$
3. (a) $2, 3, 1$
(b) $2^2 - 2(3) = -2$
(c) Each root satisfies $\alpha^3 = 2\alpha^2 - 3\alpha + 1$, so $\text{sum} = 2(-2) - 3(2) + 3 = -7$
4. $n = 1$: $12 = 6 \times 2$. Assume $7^k + 5 = 6m$. Then $7^{k+1} + 5 = 7(7^k + 5) - 30 = 42m - 30 = 6(7m - 5)$, divisible by 6. True for $n = 1$ and $k \Rightarrow k + 1$, so true for all $n \in \mathbb{Z}^+$.
5. (a) $z = 4e^{2\pi i/3}$
(b) $z^4 = 256e^{8\pi i/3} = 256e^{2\pi i/3} = -128 + 128\sqrt{3}i$
(c) $-16 = 16e^{i\pi}$, $w = 2e^{i(\pi/4 + k\pi/2)}$: $w = \pm\sqrt{2} \pm \sqrt{2}i$ (all four sign combinations)
6. (a) $\frac{1}{2}e^x + \frac{5}{2}e^{-x} = 3 \Rightarrow e^{2x} - 6e^x + 5 = 0 \Rightarrow e^x = 1$ or 5 : $x = 0$ or $x = \ln 5$
(b) $\int \frac{1}{\sqrt{(x+2)^2 + 9}} dx = \operatorname{arsinh}\left(\frac{x+2}{3}\right) + c$
7. (a) $\sum (4r^2 - 4r + 1) = \frac{4n(n+1)(2n+1)}{6} - 2n(n+1) + n = \frac{n}{3}[2(n+1)(2n+1) - 6(n+1) + 3] = \frac{n}{3}(4n^2 - 1) = \frac{n(2n-1)(2n+1)}{3}$
(b) $\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$; $\text{sum} = 1 - \frac{1}{n+1} = \frac{n}{n+1}$; $\text{sum to infinity} = 1$
8. (a) $\det B = 1(1 - 0) - 2(0 - 6) + 0 = 13$
(b) $B^{-1} = \frac{1}{13} \begin{pmatrix} 1 & -2 & 6 \\ 6 & 1 & -3 \\ -2 & 4 & 1 \end{pmatrix}$
(c) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{13} \begin{pmatrix} 3 - 8 + 30 \\ 18 + 4 - 15 \\ -6 + 16 + 5 \end{pmatrix} = \begin{pmatrix} 25/13 \\ 7/13 \\ 15/13 \end{pmatrix}$
9. (a) $2x - 2x^2 + \frac{8}{3}x^3$, valid for $-\frac{1}{2} < x \leq \frac{1}{2}$
(b) $x = 0.1$: $0.2 - 0.02 + 0.002667 = 0.1827$; calculator $\ln 1.2 = 0.1823$
10. (a) $\frac{1}{2} \int_0^{2\pi} (4 + 4\cos\theta + \cos^2\theta) d\theta = \frac{1}{2}(8\pi + \pi) = \frac{9\pi}{2}$
(b) $y = 2\sin\theta + \sin\theta\cos\theta$; $\frac{dy}{d\theta} = 2\cos\theta + \cos 2\theta = 2\cos^2\theta + 2\cos\theta - 1 = 0 \Rightarrow \cos\theta = \frac{-1 + \sqrt{3}}{2}$,
so $r = 2 + \frac{\sqrt{3} - 1}{2} = \frac{3 + \sqrt{3}}{2}$
11. (a) $\pi \int_0^{\ln 2} e^{-2x} dx = \frac{\pi}{2} \left(1 - \frac{1}{4}\right) = \frac{3\pi}{8}$
(b) $\frac{\sinh(\ln 3)}{\ln 3} = \frac{4/3}{\ln 3} = \frac{4}{3\ln 3} = 1.21$
(c) $\left[\frac{1}{2} \arctan \frac{x+1}{2}\right]_0^1 = \frac{\pi}{8} - \frac{1}{2} \arctan \frac{1}{2}$

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