

A-Level Maths – Pure Year 2

A-Level Mathematics (Edexcel/AQA/OCR), Year 2 · Total marks: 54 · Answers on the last page

Practice questions covering functions, sequences and series, radians, parametric equations, integration techniques, differential equations, and numerical methods, matching the second-year A-Level Pure Maths specification. A calculator is allowed throughout.

Section A – Skills

1. The function $f(x) = \frac{x-1}{x+2}$ is defined for $x \neq -2$.
 (a) Find $f^{-1}(x)$.
 (b) State the value of x that must be excluded from the domain of f^{-1} . [4]
2. A geometric sequence has first term $a = 5$ and common ratio $r = 1.5$.
 (a) Find the sum of the first 10 terms, S_{10} , giving your answer to 1 decimal place.
 (b) Find the number of the first term of the sequence that exceeds 200. [5]
3. A sector of a circle has radius 8 cm and angle 1.2 radians. Find:
 (a) the arc length of the sector,
 (b) the area of the sector. [3]
4. Using the small angle approximations $\sin \theta \approx \theta$ and $\cos \theta \approx 1 - \frac{\theta^2}{2}$, show that for small x , $\cos(2x) \approx 1 - 2x^2$. [3]

Section B – Exam-style

5. A curve has parametric equations $x = t^2$, $y = 2t$.
 (a) Find $\frac{dy}{dx}$ in terms of t .
 (b) Find the equation of the tangent to the curve at the point where $t = 3$. [5]
6. Use integration by parts to find $\int x e^{2x} dx$. [4]
7. Use the identity $\cos(2x) = 1 - 2\sin^2 x$ to find $\int \sin^2 x dx$. [4]
8. Express $\frac{3x+5}{(x+1)(x+2)}$ in partial fractions, and hence find $\int \frac{3x+5}{(x+1)(x+2)} dx$. [5]

Section C – Challenge (Differential equations and numerical methods)

9. Solve the differential equation $\frac{dy}{dx} = 2xy$, given that $y = 3$ when $x = 0$. Give your answer in the form $y = f(x)$. [5]
10. Show that the equation $x^3 - x - 2 = 0$ has a root between $x = 1$ and $x = 2$. Starting with $x_0 = 1.5$, use the Newton-Raphson method to find this root correct to 3 decimal places after two iterations. [5]

11. Use the trapezium rule with 4 strips (5 ordinates, $h = 0.5$) to estimate $\int_0^2 \sqrt{1+x^3} dx$, giving your answer to 3 decimal places. [5]
12. The temperature θ (in $^{\circ}\text{C}$) of a cooling liquid in a room of temperature 20°C satisfies Newton's law of cooling, $\theta = 20 + 80e^{-kt}$, where t is time in minutes. The liquid starts at 100°C and cools to 70°C after 5 minutes.
- (a) Find the value of k , correct to 3 significant figures.
- (b) Find the temperature after 15 minutes, correct to 1 decimal place. [6]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

- $f^{-1}(x) = \frac{1+2x}{1-x}$.
 - $x = 1$ must be excluded.
- $S_{10} = \frac{5(1.5^{10} - 1)}{1.5 - 1} = 566.7$.
 - The 11th term is the first to exceed 200 (it equals 288.3; the 10th term is 192.2, which does not).
- Arc length $= r\theta = 8 \times 1.2 = 9.6$ cm.
 - Area $= \frac{1}{2}r^2\theta = \frac{1}{2}(8)^2(1.2) = 38.4$ cm².
- Using $\cos(2x) \approx 1 - \frac{(2x)^2}{2} = 1 - \frac{4x^2}{2} = 1 - 2x^2$, as required.
- $\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 2$, so $\frac{dy}{dx} = \frac{1}{t}$.
 - At $t = 3$: point $(9, 6)$, gradient $\frac{1}{3}$. Tangent: $y = \frac{1}{3}x + 3$.
- $\int xe^{2x} dx = \left(\frac{x}{2} - \frac{1}{4}\right)e^{2x} + C$.
- $\sin^2 x = \frac{1 - \cos(2x)}{2}$, so $\int \sin^2 x dx = \frac{x}{2} - \frac{\sin(2x)}{4} + C$.
- $\frac{3x+5}{(x+1)(x+2)} = \frac{2}{x+1} + \frac{1}{x+2}$. Integral $= 2\ln|x+1| + \ln|x+2| + C$.
- Separating variables: $\int \frac{1}{y} dy = \int 2x dx \Rightarrow \ln|y| = x^2 + C \Rightarrow y = Ae^{x^2}$. Using $y(0) = 3$ gives $A = 3$, so $y = 3e^{x^2}$.
- $f(1) = -2 < 0$, $f(2) = 4 > 0$, so there is a sign change and hence a root between 1 and 2. With $f'(x) = 3x^2 - 1$: $x_1 = 1.5 - \frac{f(1.5)}{f'(1.5)} = 1.522$, $x_2 = 1.521$. The root is 1.521 (3 d.p.).
- Using $x = 0, 0.5, 1, 1.5, 2$ with y -values 1.000, 1.061, 1.414, 2.092, 3.000: $\int_0^2 \sqrt{1+x^3} dx \approx \frac{0.5}{2} [(1.000 + 3.000) + 2(1.061 + 1.414 + 2.092)] = 3.283$.
- $70 = 20 + 80e^{-5k} \Rightarrow e^{-5k} = 0.625 \Rightarrow k = \frac{-\ln(0.625)}{5} = 0.0940$.
 - $\theta(15) = 20 + 80e^{-0.0940 \times 15} = 39.5^\circ \text{C}$.

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