

AP Calculus AB – Practice Set

AP Calculus AB · Total marks: 71 · Answers on the last page

Covers limits and continuity, derivatives and their applications, integrals and the Fundamental Theorem, and separable differential equations. Section A is short-answer in the style of multiple-choice items (no calculator); Sections B and C are free-response style. Allow about 90 minutes.

Section A – Short answer (MCQ style) (No calculator)

1. Evaluate each limit.

(a) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$

(b) $\lim_{x \rightarrow \infty} \frac{3x^2 + 1}{2x^2 - x}$

(c) $\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$

(d) $\lim_{x \rightarrow 0^+} \frac{\ln x}{x}$

[4]

2. Find $\frac{dy}{dx}$ for each function.

(a) $y = x^2 e^{3x}$

(b) $y = \ln(\cos x)$

(c) $y = \frac{x}{x^2 + 1}$

(d) $y = \arctan(2x)$

[4]

3. The curve $x^2 + y^2 = 25$ passes through the point $(3, 4)$.

(a) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

(b) Write an equation of the line tangent to the curve at $(3, 4)$.

[3]

4. Evaluate each integral.

(a) $\int (3x^2 - 4x + 1) dx$

(b) $\int_0^{\pi/2} \cos x dx$

(c) $\int \frac{x}{x^2 + 1} dx$

(d) $\int_1^e \frac{\ln x}{x} dx$

[4]

5. Let $g(x) = \int_1^{x^2} \sin t dt$.

(a) Find $g'(x)$.

(b) Find $g'(\sqrt{\pi/2})$.

[3]

6. Let $f(x) = \frac{x^2 - 4}{x - 2}$ for $x \neq 2$ and $f(2) = k$.

(a) Find the value of k that makes f continuous at $x = 2$.

(b) With this value of k , is f differentiable at $x = 2$? Justify your answer.

(c) The function $h(x) = \frac{1}{x - 2}$ has a discontinuity at $x = 2$. State the type of discontinuity.

[4]

Section B – Free response (Calculator allowed)

7. A particle moves along the x -axis with velocity $v(t) = t^2 - 4t + 3$ for $0 \leq t \leq 5$, where t is in seconds and v in metres per second. At $t = 0$ the particle is at $x = 2$.
- (a) Find all times at which the particle is at rest.
 - (b) Find the displacement of the particle over $0 \leq t \leq 5$.
 - (c) Find the total distance travelled over $0 \leq t \leq 5$.
 - (d) Find the position of the particle at $t = 5$.
 - (e) Is the speed of the particle increasing or decreasing at $t = 2$? Justify your answer. [9]
8. A 10-foot ladder leans against a vertical wall. The bottom of the ladder slides away from the wall at a constant rate of 2 feet per second.
- (a) Find the rate at which the top of the ladder is sliding down the wall when the bottom is 6 feet from the wall.
 - (b) Find the rate of change of the area of the triangle formed by the ladder, the wall and the ground at that instant. [6]
9. An open-top box with a square base is to have a volume of 32 cubic feet. Let x be the side length of the base and h the height.
- (a) Express the surface area S of the box (base plus four sides) as a function of x alone.
 - (b) Find the dimensions that minimize the surface area, and justify that your answer gives a minimum.
 - (c) State the minimum surface area. [7]
10. Consider the differential equation $\frac{dy}{dx} = xy$ with $y(1) = 2$.
- (a) Write an equation of the line tangent to the solution curve at $(1, 2)$ and use it to approximate $y(1.2)$.
 - (b) Find the particular solution $y = f(x)$.
 - (c) Use your solution to find $y(1.2)$ to three decimal places, and state whether the tangent-line approximation is an over- or under-estimate. Justify using $\frac{d^2y}{dx^2}$. [8]

Section C – Extended free response (Calculator allowed)

11. Let R be the region in the first quadrant bounded by $y = \sqrt{x}$ and $y = \frac{x}{2}$.
- (a) Find the x -coordinates of the points of intersection.
 - (b) Find the area of R .
 - (c) Find the volume of the solid generated when R is rotated about the x -axis.
 - (d) The region R is the base of a solid whose cross-sections perpendicular to the x -axis are squares. Find the volume of this solid. [9]
12. Let $f(x) = x^3 - 6x^2 + 9x + 1$.
- (a) Find the intervals on which f is increasing and decreasing.
 - (b) Find the x -coordinates of all relative extrema and classify each.
 - (c) Find the point of inflection of the graph of f .
 - (d) Find the absolute maximum and absolute minimum values of f on $[0, 4]$.
 - (e) Verify that the Mean Value Theorem applies to f on $[0, 4]$ and find all values of c guaranteed by the theorem. [10]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

- 6
 - $\frac{3}{2}$
 - 5
 - $-\infty$
- $e^{3x}(2x + 3x^2)$
 - $-\tan x$
 - $\frac{1 - x^2}{(x^2 + 1)^2}$
 - $\frac{2}{1 + 4x^2}$
- $\frac{dy}{dx} = -\frac{x}{y}$
 - $y - 4 = -\frac{3}{4}(x - 3)$
- $x^3 - 2x^2 + x + C$
 - 1
 - $\frac{1}{2}\ln(x^2 + 1) + C$
 - $\frac{1}{2}$
- $g'(x) = 2x \sin(x^2)$
 - $2\sqrt{\pi/2} \cdot \sin(\pi/2) = \sqrt{2\pi}$
- $k = 4$
 - Yes: with $k = 4$, $f(x) = x + 2$ for all x , so $f'(2) = 1$
 - Infinite (non-removable) discontinuity; $x = 2$ is a vertical asymptote
- $t = 1$ and $t = 3$
 - $\int_0^5 v(t) dt = \frac{20}{3} \text{ m}$
 - $\frac{4}{3} + \frac{4}{3} + \frac{20}{3} = \frac{28}{3} \approx 9.33 \text{ m}$
 - $x(5) = 2 + \frac{20}{3} = \frac{26}{3}$
 - $v(2) = -1 < 0$, $a(2) = 2(2) - 4 = 0$; since $a(2) = 0$ the speed is neither increasing nor decreasing at that instant
- $x^2 + y^2 = 100 \Rightarrow \frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt} = -\frac{6}{8}(2) = -1.5 \text{ ft/s}$ (moving down at 1.5 ft/s)
 - $A = \frac{1}{2}xy$, $\frac{dA}{dt} = \frac{1}{2} \left(y \frac{dx}{dt} + x \frac{dy}{dt} \right) = \frac{1}{2}(16 - 9) = 3.5 \text{ ft}^2/\text{s}$
- $h = \frac{32}{x^2}$, $S(x) = x^2 + \frac{128}{x}$, $x > 0$
 - $S'(x) = 2x - \frac{128}{x^2} = 0 \Rightarrow x^3 = 64$, $x = 4$, $h = 2$; $S''(x) = 2 + \frac{256}{x^3} > 0$ so minimum
 - $S(4) = 16 + 32 = 48 \text{ ft}^2$
- Slope = 2: $y = 2 + 2(x - 1)$; $y(1.2) \approx 2.4$

(b) $\ln y = \frac{x^2}{2} + C, C = \ln 2 - \frac{1}{2}; y = 2e^{(x^2-1)/2}$

(c) $y(1.2) = 2e^{0.22} = 2.492$; underestimate because $\frac{d^2y}{dx^2} = y + x\frac{dy}{dx} = y + x^2y > 0$ (concave up near $x = 1$)

11. (a) $\sqrt{x} = \frac{x}{2} \Rightarrow x = 0, 4$

(b) $\int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) dx = \frac{16}{3} - 4 = \frac{4}{3}$

(c) $\pi \int_0^4 \left(x - \frac{x^2}{4}\right) dx = \pi \left(8 - \frac{16}{3}\right) = \frac{8\pi}{3}$

(d) $\int_0^4 \left(\sqrt{x} - \frac{x}{2}\right)^2 dx = 8 - \frac{64}{5} + \frac{16}{3} = \frac{8}{15}$

12. (a) $f'(x) = 3(x-1)(x-3)$: increasing on $(-\infty, 1)$ and $(3, \infty)$, decreasing on $(1, 3)$

(b) Relative maximum at $x = 1$ ($f = 5$), relative minimum at $x = 3$ ($f = 1$)

(c) $f''(x) = 6x - 12 = 0 \Rightarrow (2, 3)$

(d) $f(0) = 1, f(1) = 5, f(3) = 1, f(4) = 5$: absolute max 5, absolute min 1

(e) f is a polynomial (continuous and differentiable); $\frac{f(4) - f(0)}{4} = 1; 3c^2 - 12c + 9 = 1 \Rightarrow c = 2 \pm \frac{2\sqrt{3}}{3}$, i.e. $c \approx 0.845, 3.155$

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