

AP Physics C: Mechanics – Practice Set

AP Physics C: Mechanics (College Board) · Total marks: 44 · Answers on the last page

Calculus-based mechanics practice in AP Physics C style: kinematics via derivatives/integrals, work-energy, momentum and impulse, rotational dynamics, and oscillations. A calculator is allowed; use $g = 9.81 \text{ m s}^{-2}$. Free-response style; allow about 45 minutes.

Section A – Calculus-based kinematics and energy

- The position of a particle moving along the x -axis is given by $x(t) = 2t^3 - 3t^2 + 4$ (in metres, t in seconds).
 - Derive expressions for the velocity $v(t)$ and acceleration $a(t)$.
 - Evaluate v and a at $t = 2 \text{ s}$. [4]
- A particle moves with velocity $v(t) = 3t^2 - 12t + 9 \text{ m s}^{-1}$.
 - Find the times, for $t \geq 0$, at which the particle is momentarily at rest.
 - Find the net displacement of the particle between $t = 0 \text{ s}$ and $t = 4 \text{ s}$. [4]
- A variable force acting on an object along the x -axis is $F(x) = 3x^2 \text{ N}$ (with x in metres). Calculate the work done by this force as the object moves from $x = 0$ to $x = 4 \text{ m}$. [3]
- A block of mass 4.0 kg , initially at rest, is acted on by a time-varying net force $F(t) = 6t^2 \text{ N}$ for $0 \leq t \leq 3 \text{ s}$.
 - Calculate the impulse delivered to the block over this interval.
 - Calculate the block's velocity at $t = 3 \text{ s}$. [4]

Section B – Rotation and momentum (exam-style)

- A uniform disk of mass 2.0 kg and radius 0.30 m , free to rotate about its central axis, experiences a constant net torque of 5.0 N m .
 - Calculate the moment of inertia of the disk about its axis (use $I = \frac{1}{2}MR^2$).
 - Calculate the resulting angular acceleration. [3]
- A uniform rod of mass 3.0 kg and length 1.2 m rotates about a pivot at one end (use $I = \frac{1}{3}ML^2$ for a rod about its end) at constant angular velocity $\omega = 4.0 \text{ rad s}^{-1}$.
 - Calculate the moment of inertia of the rod about the pivot.
 - Calculate its angular momentum and rotational kinetic energy. [4]
- A uniform solid disk (mass M , radius R , $I = \frac{1}{2}MR^2$) is released from rest and rolls without slipping down a ramp, descending a vertical height $h = 2.0 \text{ m}$.
 - Using energy conservation, show that $v = \sqrt{\frac{4gh}{3}}$ for the speed of the centre of mass at the bottom.
 - Calculate this speed numerically. [5]
- Explain why the disk in the previous question reaches the bottom of the ramp with a smaller speed than an identical-mass block sliding down a frictionless ramp of the same height, even though both start from rest at the same height. [2]

Section C – Oscillations (challenge)

9. A particle undergoes simple harmonic motion with position $x(t) = 0.10 \cos(5t)$ (SI units).
(a) Derive $v(t)$ and $a(t)$ by differentiation.
(b) State the maximum speed and maximum magnitude of acceleration of the particle. [4]
10. For the oscillator in the previous question, verify by substitution that $x(t) = 0.10 \cos(5t)$ satisfies the SHM differential equation $\frac{d^2x}{dt^2} = -\omega^2x$, and state the value of ω and the period T . [3]
11. A block–spring system undergoes SHM. At a certain instant the block is at displacement $x = 0.060$ m from equilibrium and has speed 0.80 m s^{-1} ; the amplitude of the motion is 0.10 m. Using energy conservation ($\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$), and given the mass is 0.50 kg, calculate the spring constant k . [4]
12. A physical pendulum consists of a uniform rod of length $L = 0.80$ m pivoted at one end, oscillating with small amplitude. Its period is given by $T = 2\pi\sqrt{\frac{I}{Mgd}}$, where $I = \frac{1}{3}ML^2$ is the moment of inertia about the pivot and $d = L/2$ is the distance from the pivot to the centre of mass. Show that $T = 2\pi\sqrt{\frac{2L}{3g}}$ and calculate its numerical value. [4]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

- (a) $v(t) = \frac{dx}{dt} = 6t^2 - 6t$; $a(t) = \frac{dv}{dt} = 12t - 6$.

(b) At $t = 2$ s: $v = 6(4) - 6(2) = 12 \text{ m s}^{-1}$, $a = 12(2) - 6 = 18 \text{ m s}^{-2}$.
- (a) Set $v(t) = 0$: $3t^2 - 12t + 9 = 0 \Rightarrow t^2 - 4t + 3 = 0 \Rightarrow (t - 1)(t - 3) = 0$, so $t = 1$ s and $t = 3$ s.

(b) $x(t) = \int v dt = t^3 - 6t^2 + 9t (+C)$. Displacement $= x(4) - x(0) = (64 - 96 + 36) - 0 = 4$ m.
- $W = \int_0^4 F(x) dx = \int_0^4 3x^2 dx = [x^3]_0^4 = 64 \text{ J}$.
- (a) $J = \int_0^3 F(t) dt = \int_0^3 6t^2 dt = [2t^3]_0^3 = 54 \text{ kg m s}^{-1}$.

(b) By the impulse-momentum theorem, $J = \Delta p = mv$ (starts at rest), so $v = \frac{54}{4.0} = 13.5 \text{ m s}^{-1}$.
- (a) $I = \frac{1}{2}MR^2 = 0.5(2.0)(0.30)^2 = 0.090 \text{ kg m}^2$.

(b) $\alpha = \frac{\tau}{I} = \frac{5.0}{0.090} = 55.6 \text{ rad s}^{-2}$.
- (a) $I = \frac{1}{3}ML^2 = \frac{1}{3}(3.0)(1.2)^2 = 1.44 \text{ kg m}^2$.

(b) $L = I\omega = 1.44(4.0) = 5.76 \text{ kg m}^2\text{s}^{-1}$. $E_k = \frac{1}{2}I\omega^2 = \frac{1}{2}(1.44)(4.0)^2 = 11.5 \text{ J}$.
- (a) Energy conservation: $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. For rolling without slipping $\omega = v/R$, so $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}MR^2)\left(\frac{v}{R}\right)^2 = \frac{1}{4}Mv^2$. Then $Mgh = \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{3}{4}Mv^2$, so $v^2 = \frac{4gh}{3}$ and $v = \sqrt{\frac{4gh}{3}}$.

(b) $v = \sqrt{\frac{4(9.81)(2.0)}{3}} = 5.11 \text{ m s}^{-1}$.
- For the sliding block, all of the lost gravitational PE converts to translational kinetic energy ($v = \sqrt{2gh}$). For the rolling disk, some of the lost gravitational PE also goes into rotational kinetic energy (needed to spin the disk up as it rolls), leaving less energy available for translational KE, so its final translational speed is smaller.
- (a) $v(t) = \frac{dx}{dt} = -0.50 \sin(5t)$; $a(t) = \frac{dv}{dt} = -2.5 \cos(5t)$.

(b) Maximum speed $= 0.50 \text{ m s}^{-1}$ (when $\sin(5t) = \pm 1$); maximum acceleration magnitude $= 2.5 \text{ m s}^{-2}$ (when $\cos(5t) = \pm 1$).
- From (a), $a(t) = -2.5 \cos(5t) = -25[0.10 \cos(5t)] = -25x(t)$, which matches $-\omega^2 x$ with $\omega^2 = 25$, so $\omega = 5 \text{ rad s}^{-1}$ — confirming the equation is satisfied. Period $T = \frac{2\pi}{\omega} = \frac{2\pi}{5} = 1.26 \text{ s}$.
- From energy conservation: $k(A^2 - x^2) = mv^2 \Rightarrow k = \frac{mv^2}{A^2 - x^2} = \frac{0.50(0.80)^2}{(0.10)^2 - (0.060)^2} = \frac{0.320}{0.0064} = 50 \text{ N m}^{-1}$.
- Substituting $I = \frac{1}{3}ML^2$ and $d = L/2$: $T = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg(L/2)}} = 2\pi\sqrt{\frac{2L}{3g}}$. Numerically, $T = 2\pi\sqrt{\frac{2(0.80)}{3(9.81)}} = 2\pi\sqrt{0.0544} = 2\pi(0.2333) = 1.47 \text{ s}$.

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