

AP Statistics – Inference Practice

AP Statistics · Total marks: 74 · Answers on the last page

Covers confidence intervals and significance tests for proportions and means (one-sample, two-sample and paired), chi-square tests, and inference for the slope of a regression line. A graphing calculator is allowed; state conditions, name the procedure, and interpret conclusions in context. Allow about 90 minutes.

Section A – Skills (Give calculated values to 3 decimal places.)

1. In a random sample of 400 adults from a city, 120 said they cycle to work at least once a week.
 - (a) Verify that the conditions for a one-sample z -interval for a proportion are met.
 - (b) Construct and interpret a 95 percent confidence interval for the proportion of all adults in the city who cycle to work at least once a week. [6]
2. A random sample of 25 packets of coffee has mean mass 52.4 g and standard deviation 6.8 g. A dotplot of the sample shows no strong skew or outliers.
 - (a) Explain why a t -procedure is appropriate.
 - (b) Construct a 90 percent confidence interval for the mean mass of all packets. [5]
3. A polling organisation wants to estimate the proportion of voters who support a proposal to within ± 0.03 with 95 percent confidence. No prior estimate is available. Find the minimum sample size required. [3]
4. A company claims that 60 percent of its customers renew their subscription. In a random sample of 400 customers, 210 renewed.
 - (a) State the hypotheses for a test of the company's claim.
 - (b) Calculate the test statistic and P -value.
 - (c) State your conclusion at $\alpha = 0.05$. [6]
5. A machine is set to fill bottles with 500 mL. A random sample of 36 bottles has mean 492 mL and standard deviation 18 mL.
 - (a) Calculate the t statistic for testing $H_0 : \mu = 500$ against $H_a : \mu < 500$.
 - (b) Find the P -value and state a conclusion at the 1 percent significance level. [5]

Section B – Exam-style (Full four-step procedure expected where a test or interval is requested.)

6. In a randomised experiment, 150 patients received a new drug and 150 received a placebo. 45 of the drug group and 30 of the placebo group reported improvement. Carry out a test at $\alpha = 0.05$ to determine whether the drug increases the proportion of patients who improve. [7]
7. Twenty students took a reaction-time test before and after drinking a caffeinated drink. For each student the difference (before minus after, in hundredths of a second) was computed. The mean difference was 2.3 with standard deviation 4.1; a plot of the differences was roughly symmetric.
 - (a) Explain why a paired t -test is appropriate rather than a two-sample t -test.
 - (b) Test whether caffeine reduces reaction time, using $\alpha = 0.05$. [7]
8. A six-sided die is rolled 120 times. The observed frequencies of 1, 2, 3, 4, 5, 6 are 15, 22, 18, 25, 20, 20.
 - (a) State the hypotheses for a chi-square goodness-of-fit test.
 - (b) Calculate the chi-square statistic and its degrees of freedom.
 - (c) Find the P -value and state a conclusion at $\alpha = 0.10$. [6]

9. A random sample of 120 shoppers was classified by gender and preferred store layout (A, B or C). Males: A 30, B 20, C 10. Females: A 20, B 30, C 10.
- (a) Find the expected count for males preferring layout A.
 - (b) Calculate the chi-square statistic and degrees of freedom for a test of independence.
 - (c) State the P -value and conclusion at $\alpha = 0.05$. [7]
10. Computer output for a regression of exam score on hours studied ($n = 20$ students) gives slope $b = 2.45$ with standard error 0.62. Conditions for inference are satisfied.
- (a) Construct a 95 percent confidence interval for the population slope β .
 - (b) Test $H_0 : \beta = 0$ against $H_a : \beta \neq 0$ and interpret the result. [6]

Section C – Free-response (Investigative tasks.)

11. Two teaching methods were compared. Method A: $n = 30$, $\bar{x} = 78.2$, $s = 8.1$. Method B: $n = 28$, $\bar{x} = 74.5$, $s = 9.4$. Students were randomly assigned to methods.
- (a) Calculate the standard error of $\bar{x}_A - \bar{x}_B$.
 - (b) Carry out a two-sample t -test (using the conservative $df = 27$ or technology) to determine whether the methods differ in mean score, at $\alpha = 0.05$.
 - (c) Describe a Type I error in this context. [8]
12. Historically 20 percent of applicants to a programme are accepted. A new admissions officer accepts 16 of a random sample of 50 applicants from this year.
- (a) Check whether a one-proportion z -test is appropriate.
 - (b) Test whether the acceptance rate has increased, at $\alpha = 0.05$.
 - (c) Explain what is meant by the power of this test, and give one way to increase it. [8]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

- (a) Random sample; $n\hat{p} = 120 \geq 10$ and $n(1 - \hat{p}) = 280 \geq 10$; population > 4000 (10 percent condition)

(b) $0.3 \pm 1.96\sqrt{\frac{0.3(0.7)}{400}} = 0.3 \pm 0.045$, i.e. (0.255, 0.345). We are 95 percent confident the true proportion is between 0.255 and 0.345.
- (a) Population SD unknown; sample is random; $n < 30$ but the dotplot shows no strong skew or outliers, so Normality is plausible

(b) $t^* = 1.711$ (df = 24): $52.4 \pm 1.711\frac{6.8}{\sqrt{25}} = 52.4 \pm 2.33$, i.e. (50.1, 54.7) g
- $n = \left(\frac{1.96}{0.03}\right)^2 (0.5)(0.5) = 1067.1$, so $n = 1068$
- (a) $H_0 : p = 0.6$, $H_a : p \neq 0.6$

(b) $z = \frac{0.525 - 0.6}{\sqrt{0.6(0.4)/400}} = -3.06$, $P \approx 0.0022$

(c) $P < 0.05$: reject H_0 ; convincing evidence the renewal rate differs from 60 percent
- (a) $t = \frac{492 - 500}{18/\sqrt{36}} = -2.667$, df = 35

(b) $P \approx 0.0057 < 0.01$: reject H_0 ; convincing evidence the mean fill is below 500 mL
- $H_0 : p_1 = p_2$, $H_a : p_1 > p_2$; random assignment, all counts ≥ 10
 $\hat{p}_1 = 0.3$, $\hat{p}_2 = 0.2$, pooled $\hat{p} = 0.25$
 $z = \frac{0.1}{\sqrt{0.25(0.75)(2/150)}} = 2.00$, $P \approx 0.023$
 Reject H_0 : convincing evidence the drug increases the improvement rate
- (a) Each before/after pair comes from the same student, so the observations are not independent samples

(b) $H_0 : \mu_d = 0$, $H_a : \mu_d > 0$; $t = \frac{2.3}{4.1/\sqrt{20}} = 2.51$, df = 19, $P \approx 0.011$. Reject H_0 : convincing evidence caffeine reduces reaction time
- (a) H_0 : the die is fair (each face has probability $\frac{1}{6}$); H_a : at least one probability differs

(b) Expected = 20 each; $\chi^2 = \frac{25 + 4 + 4 + 25 + 0 + 0}{20} = 2.9$, df = 5

(c) $P \approx 0.715$; fail to reject H_0 ; no convincing evidence the die is unfair
- (a) $\frac{60 \times 50}{120} = 25$

(b) Expected counts 25, 25, 10 in each row; $\chi^2 = 4 \times \frac{25}{25} = 4.0$, df = 2

(c) $P \approx 0.135$; fail to reject H_0 ; no convincing evidence of an association between gender and layout preference
- (a) $t^* = 2.101$ (df = 18): $2.45 \pm 2.101(0.62) = 2.45 \pm 1.30$, i.e. (1.15, 3.75)

(b) $t = \frac{2.45}{0.62} = 3.95$, $P \approx 0.001$; reject H_0 ; convincing evidence of a linear relationship; each extra hour is associated with about 2.45 more points

11. (a) $\sqrt{\frac{8.1^2}{30} + \frac{9.4^2}{28}} = 2.31$
 (b) $H_0 : \mu_A = \mu_B, H_a : \mu_A \neq \mu_B$; $t = \frac{3.7}{2.31} = 1.60$; $P \approx 0.115$ (df ≈ 53.5) or $P \approx 0.12$ (df = 27); fail to reject H_0 ; no convincing evidence of a difference
 (c) Concluding the methods differ in mean score when in fact they do not
12. (a) Random sample; $np_0 = 10$ and $n(1 - p_0) = 40$, both ≥ 10 ; population > 500
 (b) $H_0 : p = 0.2, H_a : p > 0.2$; $\hat{p} = 0.32$; $z = \frac{0.32 - 0.2}{\sqrt{0.2(0.8)/50}} = 2.12$, $P \approx 0.017$; reject H_0 ; convincing evidence the rate has increased
 (c) Power is the probability of correctly rejecting H_0 when a specific alternative is true; increase n or α

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