

GCSE / IGCSE Maths – Geometry

GCSE/IGCSE (Higher) · Total marks: 83 · Answers on the last page

Angle facts and polygons, circle theorems, Pythagoras in 2D and 3D, area and volume (including sectors, cones, spheres and frustums), similar shapes and transformations. Section A is non-calculator; use a calculator for Sections B and C. Allow about 75 minutes.

Section A – Skills (Non-calculator)

1. (a) Work out the size of each interior angle of a regular decagon.
(b) A regular polygon has interior angles of 156° . Work out the number of sides.
(c) Two parallel lines are crossed by a transversal. One of the angles formed is 68° . Write down the size of the co-interior (allied) angle to it, giving a reason. [5]
2. (a) A right-angled triangle has shorter sides of length 7 cm and 24 cm. Work out the length of the hypotenuse.
(b) A cuboid measures 3 cm by 4 cm by 12 cm. Work out the length of the longest diagonal of the cuboid. [5]
3. (a) A trapezium has parallel sides of length 8 cm and 14 cm and a perpendicular height of 5 cm. Work out its area.
(b) A sector of a circle has radius 6 cm and angle 60° . Work out the exact area of the sector and the exact arc length, leaving your answers in terms of π . [6]
4. A cone has base radius 3 cm and perpendicular height 4 cm.
(a) Show that the slant height is 5 cm.
(b) Work out the volume of the cone, giving your answer in terms of π .
(c) Work out the curved surface area, giving your answer in terms of π . (Curved surface area of a cone = πrl .) [6]
5. The point P has coordinates $(2, 3)$.
(a) Write down the coordinates of the image of P after a rotation of 90° clockwise about the origin.
(b) Write down the coordinates of the image of P after a reflection in the line $y = x$.
(c) The point $Q(3, 2)$ is enlarged by scale factor -2 with centre $(1, 1)$. Find the coordinates of the image of Q .
(d) Describe fully the single transformation that maps triangle A to triangle B , where B is the same shape and size as A , is the same way round (not a mirror image), and every point has moved 5 units right and 2 units down. [7]
6. Points A , B and C lie on a circle with centre O . Angle $AOB = 110^\circ$ and C is on the major arc.
(a) Work out angle ACB , giving a reason.
(b) D is a point on the minor arc AB . Work out angle ADB , giving a reason.
(c) A tangent to the circle at A meets the line OA . Write down the angle between the tangent and OA . [6]

Section B – Exam-style (Calculator allowed)

7. A cuboid $ABCDEFGH$ has base $ABCD$ with $AB = 6$ cm and $BC = 8$ cm, and vertical height $CG = 5$ cm. G is vertically above C .
(a) Work out the length AC .
(b) Work out the length AG , giving your answer to 3 significant figures.

- (c) Work out the angle that AG makes with the base $ABCD$, giving your answer to 1 decimal place. [7]
8. A solid is made from a cylinder of radius 3 cm and height 8 cm with a hemisphere of radius 3 cm fixed on top.
(a) Work out the volume of the solid, giving your answer in terms of π .
(b) The solid is made of metal with density 8.9 g/cm^3 . Work out the mass of the solid to the nearest gram. (Volume of a sphere $= \frac{4}{3}\pi r^3$.) [6]
9. Two mathematically similar bottles have heights 12 cm and 18 cm.
(a) The label on the smaller bottle has area 30 cm^2 . Work out the area of the corresponding label on the larger bottle.
(b) The larger bottle holds 540 ml. Work out how much the smaller bottle holds. [5]
10. A , B and C are points on a circle with centre O . The line TA is a tangent to the circle at A . Angle $OAB = 35^\circ$.
(a) Work out angle AOB , giving a reason.
(b) Work out angle ACB , where C is on the major arc, giving a reason.
(c) Work out angle TAB (the angle between the tangent and the chord AB) and state the circle theorem that connects your answers to (b) and (c). [7]
11. A shape is made from a square of side 10 cm with a quarter circle of radius 5 cm removed from each of two adjacent corners (the quarter circles are centred at the two corners and do not overlap).
(a) Work out the area of the shape to 3 significant figures.
(b) Work out the perimeter of the shape to 3 significant figures. [6]

Section C – Challenge (Calculator allowed)

12. A solid cone has base radius 6 cm and height 12 cm. The top of the cone is removed by a cut parallel to the base, 4 cm from the vertex, leaving a frustum.
(a) Show that the radius of the circular cut is 2 cm.
(b) Work out the volume of the frustum, giving your answer in terms of π .
(c) The frustum is melted down and recast as a sphere. Work out the radius of the sphere to 3 significant figures. [8]
13. Triangle A has vertices $(1, 1)$, $(3, 1)$ and $(1, 4)$.
(a) Triangle A is reflected in the x -axis to give triangle B . Write down the vertices of B .
(b) Triangle B is then rotated 180° about the origin to give triangle C . Write down the vertices of C .
(c) Describe fully the single transformation that maps triangle A directly to triangle C .
(d) Triangle A is enlarged by scale factor 3 about the origin. Work out the area of the enlarged triangle. [8]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

- Exterior = $360 \div 10 = 36^\circ$, interior = 144°
 - Exterior = 24° , $n = 360 \div 24 = 15$
 - 112° ; co-interior angles between parallel lines add to 180°
- $\sqrt{49 + 576} = \sqrt{625} = 25$ cm
 - $\sqrt{9 + 16 + 144} = \sqrt{169} = 13$ cm
- $\frac{1}{2}(8 + 14)(5) = 55$ cm²
 - Area = $\frac{60}{360}\pi(6^2) = 6\pi$ cm²; arc = $\frac{60}{360}(2\pi)(6) = 2\pi$ cm
- $l = \sqrt{3^2 + 4^2} = 5$ cm
 - $\frac{1}{3}\pi(3^2)(4) = 12\pi$ cm³
 - $\pi(3)(5) = 15\pi$ cm²
- (3, -2)
 - (3, 2)
 - $(1 - 2(2), 1 - 2(1)) = (-3, -1)$
 - Translation by the vector with components 5 (right) and -2 (down), i.e. (5, -2) as a column vector
- 55° ; the angle at the centre is twice the angle at the circumference
 - 125° ; opposite angles of a cyclic quadrilateral $ACBD$ add to 180°
 - 90° (tangent is perpendicular to the radius)
- $\sqrt{36 + 64} = 10$ cm
 - $\sqrt{100 + 25} = \sqrt{125} = 11.2$ cm
 - $\tan \theta = \frac{5}{10}$, $\theta = 26.6^\circ$
- $\pi(3^2)(8) + \frac{1}{2} \cdot \frac{4}{3}\pi(3^3) = 72\pi + 18\pi = 90\pi$ cm³
 - $90\pi \times 8.9 = 2516.4\ldots \approx 2516$ g
- Length scale factor $\frac{3}{2}$, area factor $\frac{9}{4}$: $30 \times \frac{9}{4} = 67.5$ cm²
 - Volume factor $\frac{27}{8}$: $540 \times \frac{8}{27} = 160$ ml
- Triangle OAB is isosceles ($OA = OB$, radii), so $AOB = 180 - 2(35) = 110^\circ$
 - $ACB = 55^\circ$; angle at the centre is twice the angle at the circumference
 - $TAB = 90 - 35 = 55^\circ$; alternate segment theorem: the angle between tangent and chord equals the angle in the alternate segment
- $100 - 2 \times \frac{1}{4}\pi(5^2) = 100 - 12.5\pi = 60.7$ cm²
 - Straight edges: $10 + 10 + 5 + 5 = 30$; arcs: $2 \times \frac{1}{4}(2\pi)(5) = 5\pi$; perimeter = $30 + 5\pi = 45.7$ cm
- Similar triangles: $r = 6 \times \frac{4}{12} = 2$ cm
 - $\frac{1}{3}\pi(36)(12) - \frac{1}{3}\pi(4)(4) = 144\pi - \frac{16\pi}{3} = \frac{416\pi}{3}$ cm³ (≈ 436 cm³)

$$(c) \frac{4}{3}\pi r^3 = \frac{416\pi}{3} \Rightarrow r^3 = 104, r = 4.70 \text{ cm}$$

13. (a) $(1, -1), (3, -1), (1, -4)$
(b) $(-1, 1), (-3, 1), (-1, 4)$
(c) Reflection in the y -axis (the line $x = 0$)
(d) Area of $A = \frac{1}{2}(2)(3) = 3$; enlarged area $= 3 \times 9 = 27$ square units

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