

IB Maths AA HL – Topic 1: Number and Algebra

IB Mathematics: Analysis and Approaches HL · Total marks: 64 · Answers on the last page

Covers sequences and series, binomial expansions, complex numbers (Cartesian, polar and Euler form, De Moivre), proof by induction, systems of linear equations and partial fractions. Sections A and B are non-calculator (Paper 1 style); Section C allows a GDC. Allow about 90 minutes.

Section A – Skills (Non-calculator.)

1. Let $z = 3 - 4i$ and $w = 1 - 2i$.
 - (a) Find $|z|$.
 - (b) Find z^*w in the form $a + bi$.
 - (c) Express $\frac{3 + 4i}{w}$ in the form $a + bi$. [6]
2. Write $-1 + \sqrt{3}i$ in the form $re^{i\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$. [3]
3. Express $\frac{3x + 5}{(x + 1)(x + 3)}$ in partial fractions. [4]
4. Find the coefficient of x^3 in the expansion of $(2 - x)^7$. [3]
5. Solve $\log_2 x + \log_2(x - 2) = 3$. [4]
6. Solve the system of equations

$$\begin{aligned} x + y + z &= 6 \\ 2x - y + z &= 3 \\ x + 2y - z &= 2. \end{aligned}$$
[4]

Section B – Exam style (Non-calculator. Show full working.)

7. Prove by mathematical induction that $\sum_{r=1}^n r 2^r = (n - 1)2^{n+1} + 2$ for all $n \in \mathbb{Z}^+$. [7]
8. (a) Use De Moivre's theorem to show that $(1 + i)^{10} = 32i$.
 (b) Find the three cube roots of $8i$, giving your answers in the form $r \operatorname{cis} \theta$ with $0 \leq \theta < 2\pi$. [7]
9. Find the first four terms of the expansion of $(1 + 2x)^{-\frac{1}{2}}$ in ascending powers of x , and state the values of x for which the expansion is valid. [5]
10. The equation $z^2 - 6z + 13 = 0$ has roots α and β where $\operatorname{Im}(\alpha) > 0$.
 - (a) Find α and β .
 - (b) Find $\alpha^2 + \beta^2$.
 - (c) Find $\arg \alpha$ in the form $\arctan k$. [6]

Section C – Challenge (GDC allowed.)

11. Consider the system

$$\begin{aligned} x + y + z &= 3 \\ x + 2y + 3z &= 6 \end{aligned}$$

$$x + 3y + kz = 9$$

where $k \in \mathbb{R}$.

(a) Show that the system has a unique solution if and only if $k \neq 5$, and find that solution.

(b) Describe the solution set when $k = 5$, giving a general solution in terms of a parameter λ . [8]

12. A geometric series has first term 3 and sum to infinity 12.

(a) Find the common ratio r .

(b) Find the sum of the first five terms, S_5 .

(c) Find the least value of n for which $S_n > 11.9$. [7]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

- $|z| = 5$
 - $z^*w = (3 + 4i)(1 - 2i) = 11 - 2i$
 - $\frac{(3 + 4i)(1 + 2i)}{5} = -1 + 2i$
- $2e^{\frac{2\pi}{3}i}$
- $\frac{1}{x+1} + \frac{2}{x+3}$
- $\binom{7}{3} 2^4(-1)^3 = -560$
- $x(x-2) = 8 \Rightarrow x^2 - 2x - 8 = 0 \Rightarrow x = 4$ (reject $x = -2$)
- $x = 1, y = 2, z = 3$
- Base case $n = 1$: LHS = 2, RHS = $0 + 2 = 2$. Assume true for $n = k$. Then $\sum_{r=1}^{k+1} r2^r = (k-1)2^{k+1} + 2 + (k+1)2^{k+1} = 2k2^{k+1} + 2 = k2^{k+2} + 2 = ((k+1) - 1)2^{(k+1)+1} + 2$. Hence true for $n = k+1$; by induction true for all $n \in \mathbb{Z}^+$.
- $1 + i = \sqrt{2} \operatorname{cis} \frac{\pi}{4}$, so $(1 + i)^{10} = 2^5 \operatorname{cis} \frac{10\pi}{4} = 32 \operatorname{cis} \frac{\pi}{2} = 32i$
 - $2 \operatorname{cis} \frac{\pi}{6}, 2 \operatorname{cis} \frac{5\pi}{6}, 2 \operatorname{cis} \frac{3\pi}{2}$ (i.e. $\sqrt{3} + i, -\sqrt{3} + i, -2i$)
- $1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3 + \dots$, valid for $|x| < \frac{1}{2}$
- $\alpha = 3 + 2i, \beta = 3 - 2i$
 - $(\alpha + \beta)^2 - 2\alpha\beta = 36 - 26 = 10$
 - $\arg \alpha = \arctan \frac{2}{3}$
- Row-reduce: $y + 2z = 3, 2y + (k-1)z = 6 \Rightarrow (k-5)z = 0$. Unique solution iff $k \neq 5$: $z = 0, y = 3, x = 0$.
 - $k = 5$: infinitely many solutions (a line): $z = \lambda, y = 3 - 2\lambda, x = \lambda$.
- $r = \frac{3}{4}$
 - $S_5 = \frac{3(1 - (3/4)^5)}{1/4} = 9.15$ (exact $\frac{2343}{256}$)
 - $12(1 - (0.75)^n) > 11.9 \Rightarrow (0.75)^n < \frac{1}{120} \Rightarrow n > 16.6$, so $n = 17$

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