

IB Maths AA HL – Topic 2: Functions

IB AA HL · Total marks: 65 · Answers on the last page

Exam-style practice on HL Topic 2: polynomials, the factor and remainder theorems, sums and products of roots, modulus functions, rational functions and inequalities. Section A is non-calculator (Paper 1 style); Sections B and C may be attempted with a GDC. Allow about 75 minutes.

Section A – Core skills (Non-calculator. Show full working.)

- Let $f(x) = 2x^3 - 5x^2 - 4x + 3$.
 - Show that $(x - 3)$ is a factor of $f(x)$.
 - Hence factorise $f(x)$ completely.
 - Solve $f(x) = 0$. [5]
- When the polynomial $p(x) = x^4 - 3x^3 + ax + 2$ is divided by $(x - 2)$ the remainder is -4 . Find the value of a . [3]
- Solve the equation $|2x - 3| = 5 - x$. [4]
- Solve the inequality $\frac{x+1}{x-2} \leq 3$. [4]
- The function f is defined by $f(x) = \frac{3x-2}{x+1}$, $x \neq -1$.
 - Write down the equations of the asymptotes of the graph of $y = f(x)$.
 - Find the coordinates of the points where the graph crosses the axes.
 - Find $f^{-1}(x)$, stating its domain. [5]

Section B – Exam-style (Calculator allowed unless stated.)

- The polynomial $p(x) = x^3 + ax^2 + bx - 12$ has a factor $(x - 2)$. When $p(x)$ is divided by $(x + 1)$ the remainder is -12 .
 - Find the values of a and b .
 - Hence show that the equation $p(x) = 0$ has exactly one real root. [6]
- The equation $2x^3 - 7x^2 + kx + 6 = 0$ has roots α , β and γ .
 - Write down the value of $\alpha + \beta + \gamma$ and of $\alpha\beta\gamma$.
 - Given that one of the roots is 3, find the value of k .
 - Find the other two roots, giving your answers in exact form. [6]
- Let $g(x) = |x^2 - 4|$.
 - Describe the graph of $y = g(x)$, giving the coordinates of any points where it meets the axes and of any local maximum.
 - Solve the equation $|x^2 - 4| = 3x$.
 - Hence solve the inequality $|x^2 - 4| < 3x$. [7]
- Solve the inequality $|x - 1| > 2|x + 2|$. [4]
- Let $f(x) = \frac{x^2 - 3x + 3}{x - 2}$, $x \neq 2$.

- (a) Show that $f(x)$ can be written in the form $x + a + \frac{b}{x-2}$ and state the values of a and b .
(b) Write down the equations of the vertical and oblique asymptotes of the graph of $y = f(x)$.
(c) Find the coordinates of the stationary points of the graph.
(d) Hence state the range of f . [8]

Section C – Challenge

11. The polynomial $p(x) = x^4 + ax^3 + bx^2 - 5x + 2$ has $(x-1)^2$ as a factor.
(a) Explain why $p(1) = 0$ and $p'(1) = 0$.
(b) Find the values of a and b .
(c) Find the number of real roots of the equation $p(x) = 0$, justifying your answer. [7]
12. (a) Find the set of values of k for which $x^2 + kx + (k+3) > 0$ for all real values of x .
(b) Solve the inequality $\frac{2x}{x-1} \geq x+1$, giving the endpoints in exact form. [6]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

- $f(3) = 54 - 45 - 12 + 3 = 0$, so $(x - 3)$ is a factor.
 - $f(x) = (x - 3)(2x^2 + x - 1) = (x - 3)(2x - 1)(x + 1)$
 - $x = 3$, $x = \frac{1}{2}$, $x = -1$
- $p(2) = 16 - 24 + 2a + 2 = 2a - 6 = -4$, so $a = 1$
- Case $2x - 3 = 5 - x$: $x = \frac{8}{3}$. Case $2x - 3 = x - 5$: $x = -2$. Both satisfy $5 - x \geq 0$, so $x = -2$ or $x = \frac{8}{3}$.
- $\frac{x+1}{x-2} - 3 = \frac{7-2x}{x-2} \leq 0$; critical values $x = 2$ and $x = 3.5$. Solution: $x < 2$ or $x \geq 3.5$.
- $x = -1$ and $y = 3$
 - $(\frac{2}{3}, 0)$ and $(0, -2)$
 - $f^{-1}(x) = \frac{x+2}{3-x}$, domain $x \neq 3$
- $p(2) = 0 \Rightarrow 2a + b = 2$; $p(-1) = -12 \Rightarrow a - b = 1$; so $a = 1$, $b = 0$.
 - $p(x) = (x - 2)(x^2 + 3x + 6)$; discriminant of the quadratic is $9 - 24 < 0$, so the only real root is $x = 2$.
- $\alpha + \beta + \gamma = \frac{7}{2}$, $\alpha\beta\gamma = -3$
 - $54 - 63 + 3k + 6 = 0 \Rightarrow k = 1$
 - $2x^3 - 7x^2 + x + 6 = (x - 3)(2x^2 - x - 2)$, so $x = \frac{1 \pm \sqrt{17}}{4}$
- Graph of $y = x^2 - 4$ with the part below the x -axis reflected upward: meets the x -axis at $(\pm 2, 0)$, local maximum at $(0, 4)$, cusps at $x = \pm 2$.
 - $x^2 - 3x - 4 = 0 \Rightarrow x = 4$ (reject -1); $4 - x^2 = 3x \Rightarrow x = 1$ (reject -4). Solutions $x = 1$, $x = 4$.
 - $1 < x < 4$
- Squaring: $(x-1)^2 > 4(x+2)^2 \Rightarrow 3x^2 + 18x + 15 < 0 \Rightarrow (x+5)(x+1) < 0$. Solution: $-5 < x < -1$.
- $f(x) = x - 1 + \frac{1}{x-2}$, so $a = -1$, $b = 1$
 - $x = 2$ and $y = x - 1$
 - $f'(x) = 1 - \frac{1}{(x-2)^2} = 0 \Rightarrow x = 1$ or $x = 3$; local maximum $(1, -1)$, local minimum $(3, 3)$
 - $f(x) \leq -1$ or $f(x) \geq 3$
- $x = 1$ is a repeated root, so $p(x) = (x - 1)^2 q(x)$ gives $p(1) = 0$ and $p'(x) = 2(x - 1)q(x) + (x - 1)^2 q'(x)$ gives $p'(1) = 0$.
 - $p(1) = 0 \Rightarrow a + b = 2$; $p'(1) = 4 + 3a + 2b - 5 = 0 \Rightarrow 3a + 2b = 1$; so $a = -3$, $b = 5$.
 - $p(x) = (x - 1)^2(x^2 - x + 2)$; discriminant $1 - 8 < 0$, so $x = 1$ is the only real root (one distinct real root, of multiplicity 2).
- $k^2 - 4(k + 3) < 0 \Rightarrow (k - 6)(k + 2) < 0 \Rightarrow -2 < k < 6$
 - $\frac{2x - (x+1)(x-1)}{x-1} \geq 0 \Rightarrow \frac{x^2 - 2x - 1}{x-1} \leq 0$; critical values $1 \pm \sqrt{2}$ and $x = 1$. Solution: $x \leq 1 - \sqrt{2}$ or $1 < x \leq 1 + \sqrt{2}$.

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