

IB Maths AA HL – Topic 5: Calculus

IB AA HL · Total marks: 64 · Answers on the last page

HL-only calculus: implicit differentiation, related rates, integration by parts, substitution and partial fractions, Maclaurin series, L'Hôpital's rule, Euler's method and first-order differential equations. Section A is non-calculator; allow about 90 minutes.

Section A – Skills (Non-calculator)

1. The curve C has equation $x^2 + xy + y^2 = 7$. Find the gradient of C at the point $(1, 2)$. [4]
2. Find $\int x e^{2x} dx$. [4]
3. Using the substitution $u = 1 + x^2$, or otherwise, find the exact value of $\int_0^1 x\sqrt{1+x^2} dx$. [5]
4. Find $\int \frac{3x+1}{(x-1)(x+2)} dx$. [5]
5. Find the Maclaurin series of $e^x \sin x$ up to and including the term in x^3 . [4]
6. Solve the differential equation $\frac{dy}{dx} = xy$, given that $y = 2$ when $x = 0$. Give your answer in the form $y = f(x)$. [4]

Section B – Exam-style (Calculator allowed)

7. A spherical balloon is inflated so that its radius increases at a constant rate of 0.5 cm s^{-1} .
 - (a) Find the rate of increase of the volume when the radius is 4 cm.
 - (b) Find the rate of increase of the surface area at the same instant. [5]
8. A ladder of length 5 m rests against a vertical wall. The foot of the ladder slides away from the wall at 0.3 m s^{-1} . Find the rate at which the top of the ladder is sliding down the wall when the foot is 3 m from the wall. [5]
9. Consider the differential equation $\frac{dy}{dx} + \frac{2y}{x} = x$, $x > 0$, with $y = 1$ when $x = 1$.
 - (a) Find the integrating factor.
 - (b) Hence find y in terms of x . [6]
10. Consider $\frac{dy}{dx} = x + y$ with $y(0) = 1$.
 - (a) Use Euler's method with step length $h = 0.1$ to estimate $y(0.2)$.
 - (b) The exact solution is $y = 2e^x - x - 1$. Find the percentage error in your estimate. [6]

Section C – Challenge

11. (a) Find the Maclaurin series of $\ln(1+2x)$ up to and including the term in x^3 .
 (b) Use it to find an approximation for $\ln 1.2$, and compare with the calculator value.
 (c) Use l'Hôpital's rule to find $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$. [8]

12. Consider $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$, $x > 0$, $y > 0$, with $y = 2$ when $x = 1$.

(a) Use the substitution $y = vx$ to show that $v \frac{dv}{dx} = \frac{1}{x}$.

(b) Hence find y^2 in terms of x .

[8]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

1. $2x + y + x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$; at $(1, 2)$: $4 + 5 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{4}{5}$
2. $\frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C$
3. $\left[\frac{1}{3}(1+x^2)^{3/2}\right]_0^1 = \frac{2\sqrt{2}-1}{3}$
4. $\frac{3x+1}{(x-1)(x+2)} = \frac{4/3}{x-1} + \frac{5/3}{x+2}$; integral $= \frac{4}{3} \ln|x-1| + \frac{5}{3} \ln|x+2| + C$
5. $\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)\left(x-\frac{x^3}{6}\right) = x+x^2+\frac{x^3}{3}+\dots$
6. $\ln|y| = \frac{x^2}{2} + c$; $y = 2e^{x^2/2}$
7. (a) $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi(16)(0.5) = 32\pi \approx 101 \text{ cm}^3\text{s}^{-1}$
 (b) $\frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 16\pi \approx 50.3 \text{ cm}^2\text{s}^{-1}$
8. $x^2 + y^2 = 25 \Rightarrow 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$; with $x = 3$, $y = 4$: $\frac{dy}{dt} = -\frac{3(0.3)}{4} = -0.225 \text{ m s}^{-1}$ (sliding down at 0.225 m s^{-1})
9. (a) $e^{\int 2/x dx} = x^2$
 (b) $\frac{d}{dx}(x^2y) = x^3 \Rightarrow x^2y = \frac{x^4}{4} + c$; $c = \frac{3}{4}$, so $y = \frac{x^2}{4} + \frac{3}{4x^2}$
10. (a) $y_1 = 1 + 0.1(0+1) = 1.1$; $y_2 = 1.1 + 0.1(0.1+1.1) = 1.22$
 (b) Exact $y(0.2) = 1.2428$; error $= \frac{1.2428 - 1.22}{1.2428} \times 100 \approx 1.84\%$
11. (a) $2x - 2x^2 + \frac{8}{3}x^3 + \dots$
 (b) $x = 0.1$: $0.2 - 0.02 + 0.00267 = 0.1827$; calculator gives 0.1823 (agree to 3 sf)
 (c) $\lim_{x \rightarrow 0} \frac{e^x - 1}{2x} = \lim_{x \rightarrow 0} \frac{e^x}{2} = \frac{1}{2}$
12. (a) $v + x \frac{dv}{dx} = \frac{1+v^2}{v} \Rightarrow x \frac{dv}{dx} = \frac{1}{v}$, so $v \frac{dv}{dx} = \frac{1}{x}$
 (b) $\frac{v^2}{2} = \ln x + c$; $v = 2$ at $x = 1 \Rightarrow c = 2$; $y^2 = 2x^2(\ln x + 2)$

Need this topic taught, not just practised? Book a trial lesson at 50% off with Mr. Ahmad — online worldwide or in person in Dubai.

WhatsApp +971 55 373 5695 · mathandsciencetutor.com