

IB Maths AA SL – Topic 1: Number and Algebra

IB AA SL · Total marks: 64 · Answers on the last page

Practice on arithmetic and geometric sequences and series, exponents and logarithms, the binomial theorem and simple deductive proof, in the style of IB AA SL Paper 1 and Paper 2. Section A is non-calculator; allow about 75 minutes for the full set.

Section A – Skills (Non-calculator)

1. An arithmetic sequence has first term 7 and common difference 4.
 - (a) Find the 20th term.
 - (b) Find the sum of the first 20 terms. [4]
2. A geometric sequence has $u_1 = 3$ and $u_2 = 6$.
 - (a) Write down the common ratio.
 - (b) Find u_8 .
 - (c) Find the sum of the first 8 terms. [5]
3. Solve for x .
 - (a) $2^{3x-1} = 16$
 - (b) $\log_x 49 = 2$
 - (c) $\log_2 40 - \log_2 5 = x$ [6]
4. (a) Find the coefficient of x^3 in the expansion of $(2 + x)^5$.
(b) Find the constant term in the expansion of $\left(x - \frac{2}{x}\right)^6$. [5]
5. The geometric series $18 + 12 + 8 + \dots$ converges.
 - (a) Find the common ratio and state why the series converges.
 - (b) Find the sum to infinity. [4]
6. Evaluate $\sum_{k=1}^{20} (3k + 2)$. [3]

Section B – Exam-style (Calculator allowed)

7. Maya invests 5000 USD in an account paying 4.2% per annum, compounded quarterly.
 - (a) Find the value of the investment after 6 years.
 - (b) Find how many years it takes for the investment to double in value. [6]
8. In an arithmetic sequence, $u_4 = 17$ and $u_{10} = 41$.
 - (a) Find the common difference and the first term.
 - (b) Find the least value of n for which $S_n > 1000$. [6]
9. A geometric sequence has $u_2 = 6$ and $u_5 = 48$.
 - (a) Find the common ratio and the first term.
 - (b) Find S_{10} .
 - (c) Find the smallest n such that $u_n > 10\,000$. [7]
10. Solve $\log_2 x + \log_2(x - 2) = 3$. [5]

Section C – Challenge

11. In the expansion of $(1 + 2x)^n$, where $n \in \mathbb{Z}^+$, the coefficient of x^2 is 112.
(a) Show that $n = 8$.
(b) Hence find the coefficient of x^3 . [6]
12. (a) Prove that $(n + 3)^2 - (n - 3)^2 = 12n$ for all $n \in \mathbb{Z}$.
(b) Prove that the sum of the squares of any two consecutive odd integers is 2 more than a multiple of 8. [7]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

- $u_{20} = 7 + 19(4) = 83$
 - $S_{20} = \frac{20}{2}(7 + 83) = 900$
- $r = 2$
 - $u_8 = 3 \times 2^7 = 384$
 - $S_8 = \frac{3(2^8 - 1)}{2 - 1} = 765$
- $3x - 1 = 4 \Rightarrow x = \frac{5}{3}$
 - $x^2 = 49 \Rightarrow x = 7$ (base must be positive)
 - $\log_2 8 = 3$, so $x = 3$
- $\left(\frac{5}{3}\right)2^2 = 10 \times 4 = 40$
 - Term $\left(\frac{6}{3}\right)x^3 \left(-\frac{2}{x}\right)^3 = 20(-8) = -160$
- $r = \frac{2}{3}$; converges because $|r| < 1$
 - $S_\infty = \frac{18}{1 - \frac{2}{3}} = 54$
- Arithmetic series with $u_1 = 5$, $u_{20} = 62$: $S_{20} = \frac{20}{2}(5 + 62) = 670$
- $5000 \left(1 + \frac{0.042}{4}\right)^{24} = 6424.53$ USD (accept 6420 USD)
 - $(1.0105)^{4n} = 2 \Rightarrow n = \frac{\ln 2}{4 \ln 1.0105} = 16.6$ years (16.58; so during the 17th year)
- $6d = 24 \Rightarrow d = 4$, $u_1 = 5$
 - $S_n = \frac{n}{2}(10 + 4(n - 1)) = n(2n + 3)$; $2n^2 + 3n - 1000 > 0 \Rightarrow n > 21.6$, so $n = 22$ ($S_{22} = 1034$, $S_{21} = 945$)
- $r^3 = 8 \Rightarrow r = 2$, $u_1 = 3$
 - $S_{10} = 3(2^{10} - 1) = 3069$
 - $3 \times 2^{n-1} > 10000 \Rightarrow n - 1 > \log_2(3333.3) = 11.7$, so $n = 13$
- $\log_2(x(x - 2)) = 3 \Rightarrow x^2 - 2x - 8 = 0 \Rightarrow (x - 4)(x + 2) = 0$; $x = 4$ only ($x = -2$ rejected since $x > 2$ required)
- $\left(\frac{n}{2}\right)2^2 = 112 \Rightarrow \frac{n(n-1)}{2} = 28 \Rightarrow n^2 - n - 56 = 0 \Rightarrow (n - 8)(n + 7) = 0$, so $n = 8$
 - $\left(\frac{8}{3}\right)2^3 = 56 \times 8 = 448$
- LHS $= n^2 + 6n + 9 - (n^2 - 6n + 9) = 12n = \text{RHS}$
 - Let the integers be $2k + 1$ and $2k + 3$: $(2k + 1)^2 + (2k + 3)^2 = 8k^2 + 16k + 10 = 8(k^2 + 2k + 1) + 2$, which is 2 more than a multiple of 8

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