

# IB Maths AA SL – Topic 3: Geometry and Trigonometry

IB AA SL · Total marks: 62 · Answers on the last page

Practice on radians, arcs and sectors, the sine and cosine rules, trigonometric identities and equations, and 3D geometry. Section A is non-calculator; Sections B and C allow a GDC. Allow about 75 minutes. Give answers to 3 significant figures unless exact values are requested.

## Section A – Skills (Non-calculator. Exact answers required.)

- Convert  $150^\circ$  to radians, giving your answer in terms of  $\pi$ .
  - Convert  $\frac{7\pi}{4}$  radians to degrees.
  - Write down the exact value of  $\cos\left(\frac{5\pi}{6}\right)$ . [4]
- A sector of a circle has radius 6 cm and angle 1.2 radians at the centre.
  - Find the length of the arc.
  - Find the area of the sector.
  - Find the perimeter of the sector. [4]
- Given that  $\tan \theta = -2$  and  $\frac{\pi}{2} < \theta < \pi$ , find the exact values of
  - $\sin \theta$  and  $\cos \theta$ ,
  - $\sin 2\theta$ ,
  - $\cos 2\theta$ . [5]
- Show that  $\frac{1 - \cos 2x}{\sin 2x} \equiv \tan x$ , for  $\sin 2x \neq 0$ . [3]
- Solve  $2\cos^2 x + 3\sin x = 3$  for  $0 \leq x \leq 2\pi$ . Give exact answers. [5]

## Section B – Exam-style (Calculator allowed.)

- In triangle  $ABC$ ,  $AB = 8$  cm,  $AC = 11$  cm and  $\hat{A} = 52^\circ$ .
  - Find the length of  $BC$ .
  - Find the area of the triangle. [5]
- In triangle  $PQR$ ,  $PQ = 7$  cm,  $\hat{P} = 65^\circ$  and  $\hat{R} = 40^\circ$ .
  - Find  $\hat{Q}$ .
  - Find the length of  $QR$ . [4]
- Solve  $\sin\left(2x - \frac{\pi}{3}\right) = \frac{1}{2}$  for  $0 \leq x \leq \pi$ . [5]
- A cuboid  $ABCDEFGH$  has base  $ABCD$  with  $AB = 6$  cm and  $BC = 4$  cm, and vertical height  $AE = 3$  cm. Vertex  $G$  is vertically above  $C$ .
  - Find the length of the base diagonal  $AC$ .
  - Find the length of the space diagonal  $AG$ .
  - Find the angle that  $AG$  makes with the base  $ABCD$ . [6]
- A solid right cone has base radius 5 cm and slant height 13 cm.
  - Show that the vertical height of the cone is 12 cm.

- (b) Find the volume of the cone.
- (c) Find the total surface area of the cone. [6]

### Section C – Challenge (Calculator allowed.)

- 11. A chord  $AB$  of length 10 cm is drawn in a circle with centre  $O$  and radius 7 cm.
  - (a) Find the angle  $\hat{AOB}$  in radians.
  - (b) Find the area of the minor segment cut off by the chord.
  - (c) Find the perimeter of the minor segment. [7]
- 12. In triangle  $ABC$ ,  $a = BC = 8$  cm,  $b = AC = 10$  cm and  $\hat{A} = 50^\circ$ .
  - (a) Show that there are two possible triangles, and find both possible values of  $\hat{B}$ .
  - (b) For each triangle, find the length of  $AB$ .
  - (c) Find the difference between the areas of the two triangles. [8]

## Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

- $\frac{5\pi}{6}$
  - $315^\circ$
  - $-\frac{\sqrt{3}}{2}$
- 7.2 cm
  - $21.6 \text{ cm}^2$
  - 19.2 cm
- $\sin \theta = \frac{2}{\sqrt{5}}, \cos \theta = -\frac{1}{\sqrt{5}}$
  - $\sin 2\theta = -\frac{4}{5}$
  - $\cos 2\theta = -\frac{3}{5}$
- $1 - \cos 2x = 2 \sin^2 x$  and  $\sin 2x = 2 \sin x \cos x$ , so the quotient is  $\frac{2 \sin^2 x}{2 \sin x \cos x} = \frac{\sin x}{\cos x} = \tan x$ .
- Using  $\cos^2 x = 1 - \sin^2 x$ :  $2 \sin^2 x - 3 \sin x + 1 = 0$ , so  $\sin x = 1$  or  $\sin x = \frac{1}{2}$ .  
 $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$
- $BC^2 = 8^2 + 11^2 - 2(8)(11) \cos 52^\circ$ , so  $BC = 8.75 \text{ cm}$
  - Area  $= \frac{1}{2}(8)(11) \sin 52^\circ = 34.7 \text{ cm}^2$
- $\hat{Q} = 75^\circ$
  - $\frac{QR}{\sin 65^\circ} = \frac{7}{\sin 40^\circ}$ , so  $QR = 9.87 \text{ cm}$
- $2x - \frac{\pi}{3} = \frac{\pi}{6}$  or  $\frac{5\pi}{6}$  (the range  $-\frac{\pi}{3} \leq 2x - \frac{\pi}{3} \leq \frac{5\pi}{3}$  gives no others).  
 $x = \frac{\pi}{4}, \frac{7\pi}{12}$
- $AC = \sqrt{52} = 7.21 \text{ cm}$
  - $AG = \sqrt{61} = 7.81 \text{ cm}$
  - $\tan \theta = \frac{3}{\sqrt{52}}$ , so  $\theta = 22.6^\circ$
- $h = \sqrt{13^2 - 5^2} = \sqrt{144} = 12 \text{ cm}$
  - $V = \frac{1}{3}\pi(5^2)(12) = 100\pi = 314 \text{ cm}^3$
  - $A = \pi(5)(13) + \pi(5^2) = 90\pi = 283 \text{ cm}^2$
- $\sin\left(\frac{\theta}{2}\right) = \frac{5}{7}$ , so  $\theta = 1.59$  radians
  - Area  $= \frac{1}{2}(7^2)(\theta - \sin \theta) = 14.5 \text{ cm}^2$
  - Arc  $= 7\theta = 11.1 \text{ cm}$ , so perimeter  $= 21.1 \text{ cm}$
- $\sin B = \frac{10 \sin 50^\circ}{8} = 0.958 < 1$ ; since  $b > a$ , both  $B = 73.2^\circ$  and  $B = 106.8^\circ$  give  $A + B < 180^\circ$ .
  - $C = 56.8^\circ$  gives  $AB = 8.73 \text{ cm}$ ;  $C = 23.2^\circ$  gives  $AB = 4.12 \text{ cm}$
  - Areas  $\frac{1}{2}(10)(c) \sin 50^\circ$ :  $33.5 \text{ cm}^2$  and  $15.8 \text{ cm}^2$ ; difference  $17.7 \text{ cm}^2$

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