

IB Maths AA SL – Topic 5: Calculus

IB AA SL · Total marks: 59 · Answers on the last page

Practice covering differentiation (power, chain, product and quotient rules), stationary points, tangents and normals, integration, areas, kinematics and optimisation from Topic 5 of the IB AA SL syllabus. Sections A and B are non-calculator; a GDC may be used in Section C. Allow about 75 minutes.

Section A – Core skills (Non-calculator)

1. Let $f(x) = 3x^4 - 2x^2 + 5x - 1$. Find $f'(x)$. [2]
2. The curve C has equation $y = x^3 - 4x$.
 - (a) Find $\frac{dy}{dx}$.
 - (b) Find the equation of the tangent to C at the point where $x = 2$. [4]
3. Find $\int \left(6x^2 - \frac{4}{x^2} + 3 \right) dx$. [3]
4. Differentiate with respect to x :
 - (a) $y = (2x^2 - 3)^5$
 - (b) $y = x^2 \ln x$ [4]
5. Evaluate $\int_1^4 \left(2x + \frac{1}{\sqrt{x}} \right) dx$. [3]

Section B – Exam-style questions (Non-calculator)

6. The function f is defined by $f(x) = x^3 - 6x^2 + 9x + 1$.
 - (a) Find $f'(x)$ and hence find the coordinates of the two stationary points of f .
 - (b) Use the second derivative to determine the nature of each stationary point. [6]
7. A particle moves in a straight line. Its velocity at time t seconds is $v(t) = 6t - t^2 \text{ m s}^{-1}$ for $0 \leq t \leq 8$. The particle starts at the origin.
 - (a) Find the acceleration of the particle when $t = 2$.
 - (b) Find the displacement of the particle from the origin when $t = 6$.
 - (c) Find the total distance travelled by the particle in the first 8 seconds. [7]
8. The curve $y = x^2$ and the line $y = 2x + 3$ intersect at two points P and Q .
 - (a) Find the x -coordinates of P and Q .
 - (b) Find the exact area of the region enclosed between the line and the curve. [5]
9. The curve C has equation $y = \frac{2x+1}{x-2}$, $x \neq 2$.
 - (a) Show that $\frac{dy}{dx} = \frac{-5}{(x-2)^2}$.
 - (b) Find the equation of the normal to C at the point where $x = 3$. Give your answer in the form $ax + by + c = 0$ where $a, b, c \in \mathbb{Z}$. [5]

Section C – Extended problems (GDC allowed)

10. An open box is made from a square sheet of card of side 30 cm by cutting a square of side x cm from each corner and folding up the sides.
- (a) Show that the volume of the box is $V = 4x^3 - 120x^2 + 900x$.
 - (b) Find $\frac{dV}{dx}$ and hence find the value of x that gives the maximum volume.
 - (c) Find the maximum volume. [7]
11. Using the substitution $u = x^2 + 1$, or otherwise, find the exact value of $\int_0^{\sqrt{3}} x\sqrt{x^2 + 1} dx$. [4]
12. Let $f(x) = xe^{-x}$ for $x \geq 0$.
- (a) Show that $f'(x) = e^{-x}(1 - x)$.
 - (b) Find the coordinates of the maximum point of the graph of f .
 - (c) Show that $f''(x) = e^{-x}(x - 2)$ and hence find the x -coordinate of the point of inflexion.
 - (d) Find the equation of the tangent to the graph of f at the point where $x = 2$. Give your answer in the form $y = mx + c$, with m and c in exact form. [9]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

- $f'(x) = 12x^3 - 4x + 5$
- (a) $\frac{dy}{dx} = 3x^2 - 4$
(b) At $x = 2$: $y = 0$, gradient = 8; tangent $y = 8x - 16$
- $2x^3 + \frac{4}{x} + 3x + C$
- (a) $\frac{dy}{dx} = 20x(2x^2 - 3)^4$
(b) $\frac{dy}{dx} = 2x \ln x + x$
- $[x^2 + 2\sqrt{x}]_1^4 = 20 - 3 = 17$
- (a) $f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$; stationary points $(1, 5)$ and $(3, 1)$
(b) $f''(x) = 6x - 12$; $f''(1) = -6 < 0$ so $(1, 5)$ is a local maximum; $f''(3) = 6 > 0$ so $(3, 1)$ is a local minimum
- (a) $a = 6 - 2t$, so $a(2) = 2 \text{ m s}^{-2}$
(b) $s = \int_0^6 (6t - t^2) dt = \left[3t^2 - \frac{t^3}{3}\right]_0^6 = 36 \text{ m}$
(c) $v = 0$ at $t = 6$; $\int_6^8 v dt = -\frac{44}{3}$, so distance = $36 + \frac{44}{3} = \frac{152}{3} \approx 50.7 \text{ m}$
- (a) $x^2 - 2x - 3 = 0 \Rightarrow x = -1$ and $x = 3$
(b) Area = $\int_{-1}^3 (2x + 3 - x^2) dx = \left[x^2 + 3x - \frac{x^3}{3}\right]_{-1}^3 = \frac{32}{3}$
- (a) Quotient rule: $\frac{2(x-2) - (2x+1)}{(x-2)^2} = \frac{-5}{(x-2)^2}$
(b) At $x = 3$: $y = 7$, gradient of tangent = -5 , gradient of normal = $\frac{1}{5}$; normal: $y - 7 = \frac{1}{5}(x - 3)$, i.e. $x - 5y + 32 = 0$
- (a) $V = x(30 - 2x)^2 = x(900 - 120x + 4x^2) = 4x^3 - 120x^2 + 900x$
(b) $\frac{dV}{dx} = 12x^2 - 240x + 900 = 12(x-5)(x-15)$; $x = 5$ ($x = 15$ gives $V = 0$; $V'' = 24x - 240 < 0$ at $x = 5$ so maximum)
(c) $V_{\max} = 5 \times 20^2 = 2000 \text{ cm}^3$
- $\frac{1}{2} \int_1^4 u^{1/2} du = \left[\frac{2}{3}u^{3/2}\right]_1^4 = \frac{8-1}{3} = \frac{7}{3}$
- (a) Product rule: $f'(x) = e^{-x} + x(-e^{-x}) = e^{-x}(1-x)$
(b) $f'(x) = 0 \Rightarrow x = 1$; maximum at $(1, \frac{1}{e}) \approx (1, 0.368)$
(c) $f''(x) = -e^{-x}(1-x) - e^{-x} = e^{-x}(x-2)$; sign change at $x = 2$, so point of inflexion at $x = 2$
(d) $f(2) = 2e^{-2}$, $f'(2) = -e^{-2}$; tangent $y = -e^{-2}x + 4e^{-2}$

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