

IB Maths AI HL – Topic 3: Geometry and Trigonometry

IB AI HL · Total marks: 66 · Answers on the last page

Exam-style practice on the HL parts of Topic 3: vectors (including scalar and vector products and vector equations of lines), graph theory, adjacency matrices, minimum spanning trees and the Chinese postman problem. A graphic display calculator is allowed throughout; allow about 90 minutes.

Section A – Vectors (Give answers to 3 significant figures unless exact.)

- Let $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$.
 - Find $\mathbf{a} + 2\mathbf{b}$.
 - Find $|\mathbf{a}|$.
 - Find $\mathbf{a} \cdot \mathbf{b}$. [5]
- Using the vectors $\mathbf{a}$ and $\mathbf{b}$ from question 1, find the angle between $\mathbf{a}$ and $\mathbf{b}$, giving your answer in degrees. [3]
- The points $A(1, 2, -1)$ and $B(3, -1, 4)$ lie on the line L .
 - Write down a vector equation of L in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$.
 - Determine whether the point $C(7, -7, 14)$ lies on L . [5]
- Using $\mathbf{a}$ and $\mathbf{b}$ from question 1:
 - Find $\mathbf{a} \times \mathbf{b}$.
 - Hence find the area of the triangle with two sides represented by $\mathbf{a}$ and $\mathbf{b}$. [5]
- A drone's position (in metres) t seconds after launch is given by $\mathbf{r}(t) = \begin{pmatrix} 3 \\ -2 \\ 10 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \\ -0.5 \end{pmatrix}$, where the third component is height above the ground.
 - Write down the velocity vector of the drone.
 - Find the speed of the drone.
 - Find the time at which the drone reaches the ground and its position at that moment. [6]

Section B – Graph theory

- A simple graph G has vertices A, B, C, D and edges AB, AC, BC, BD and CD .
 - Write down the adjacency matrix $\mathbf{M}$ of G (vertices in alphabetical order).
 - Write down the degree of each vertex.
 - State, with a reason, whether G is Eulerian, semi-Eulerian or neither. [6]
- For the graph G in question 6, use the adjacency matrix to find:
 - the number of walks of length 2 from A to D ;
 - the number of walks of length 3 from A back to A .
 List the walks in part (a). [5]
- A weighted graph has vertices A, B, C, D, E and the following edges with weights: $AB = 5$, $AC = 3$, $AD = 6$, $BC = 4$, $BD = 2$, $CD = 7$, $CE = 5$, $DE = 3$.
 - Use Kruskal's algorithm to find a minimum spanning tree, listing the edges in the order they

are added and stating any edges rejected.

(b) State the total weight of the minimum spanning tree. [6]

9. Using the same weighted graph as question 8, apply Prim's algorithm starting from vertex A . List the edges in the order they are added and confirm the total weight. [4]

10. A complete graph K_n has n vertices.

(a) Write down the number of edges in K_5 .

(b) Explain why K_5 is Eulerian.

(c) Find the number of distinct Hamiltonian cycles in K_5 .

(d) A tree has 12 vertices. State the number of edges it has. [5]

Section C – Chinese postman problem

11. A road inspector must drive along every road in a village and return to the start. The roads are represented by a weighted graph with vertices A, B, C, D and edges $AB = 6$, $AC = 4$, $BC = 5$, $BD = 7$, $CD = 3$ (lengths in km).

(a) Find the total length of all the roads.

(b) Identify the vertices of odd degree.

(c) Find the length of the shortest route that covers every road and returns to the starting vertex, stating which road must be repeated. [6]

12. A snow plough must clear every street in a network with vertices P, Q, R, S, T . Edges and lengths (in km): $PQ = 8$, $PR = 5$, $PS = 9$, $QR = 6$, $QS = 7$, $RT = 4$, $ST = 3$, $QT = 10$.

(a) Show that the total length of the network is 52 km.

(b) List the vertices of odd degree.

(c) By considering all possible pairings of the odd vertices, find the minimum total length of the edges that must be repeated.

(d) Hence find the length of the shortest closed route that clears every street, starting and finishing at P .

(e) The plough may instead start at one odd vertex and finish at another. Determine which two vertices should be the start and finish to minimise the route, and state the new minimum length.

[10]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

1. (a) $\begin{pmatrix} 4 \\ 7 \\ -1 \end{pmatrix}$
 (b) $\sqrt{14} = 3.74$
 (c) $2 - 4 - 6 = -8$
2. $\cos \theta = \frac{-8}{\sqrt{14}\sqrt{21}} = -0.467$, so $\theta = 118^\circ$ (117.8^{circ}).
3. (a) $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$
 (b) $\lambda = 3$ satisfies all three components ($1 + 6 = 7$, $2 - 9 = -7$, $-1 + 15 = 14$), so C lies on L .
4. (a) $\begin{pmatrix} -10 \\ 7 \\ 9 \end{pmatrix}$
 (b) $\text{Area} = \frac{1}{2}\sqrt{100 + 49 + 81} = \frac{1}{2}\sqrt{230} = 7.58 \text{ units}^2$
5. (a) $\begin{pmatrix} 4 \\ 1 \\ -0.5 \end{pmatrix} \text{ m s}^{-1}$
 (b) $\sqrt{16 + 1 + 0.25} = \sqrt{17.25} = 4.15 \text{ m s}^{-1}$
 (c) $10 - 0.5t = 0 \Rightarrow t = 20 \text{ s}$; position $(83, 18, 0)$
6. (a) $\mathbf{M} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$
 (b) $\deg A = 2$, $\deg B = 3$, $\deg C = 3$, $\deg D = 2$
 (c) Semi-Eulerian: exactly two vertices (B and C) have odd degree, so an Eulerian trail exists but not an Eulerian circuit.
7. (a) $(\mathbf{M}^2)_{AD} = 2$: $A-B-D$ and $A-C-D$
 (b) $(\mathbf{M}^3)_{AA} = 2$ ($A-B-C-A$ and $A-C-B-A$)
8. (a) Add BD (2), AC (3), DE (3), BC (4); reject AB (5), CE (5), AD (6), CD (7) as each would form a cycle.
 (b) Total weight $= 2 + 3 + 3 + 4 = 12$
9. AC (3), BC (4), BD (2), DE (3); total $= 12$ (same weight as Kruskal, as expected).
10. (a) $\binom{5}{2} = 10$
 (b) Every vertex has degree 4 (even) and the graph is connected.
 (c) $\frac{(5-1)!}{2} = 12$
 (d) 11
11. (a) 25 km
 (b) B (degree 3) and C (degree 3)
 (c) Shortest B to C path is $BC = 5$ (direct; $B-D-C = 10$). Route length $= 25 + 5 = 30 \text{ km}$, repeating BC .

12. (a) $8 + 5 + 9 + 6 + 7 + 4 + 3 + 10 = 52$ km
(b) P (3), R (3), S (3), T (3); Q has degree 4
(c) Pairings: $PR + ST = 5 + 3 = 8$; $PS + RT = 9 + 4 = 13$; $PT + RS = 9 + 7 = 16$ (shortest paths P - R - T and R - T - S). Minimum = 8 km.
(d) $52 + 8 = 60$ km
(e) Only one pair of odd vertices now needs to be joined by repeated edges; choose the pair with the shortest connection, $ST = 3$, and start/finish at the other two. Start at P and finish at R (or vice versa); minimum length = $52 + 3 = 55$ km.

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