

IB Maths AI HL – Topic 4: Statistics and Probability

IB Mathematics: Applications & Interpretation HL · Total marks: 47 · Answers on the last page

Practice questions on the Poisson distribution, transition matrices, hypothesis testing and confidence intervals from IB AI HL Topic 4. A GDC is required throughout; give answers to three significant figures unless stated otherwise.

Section A – Poisson Distribution (Calculator (GDC) required)

- The number of emails, X , received by an office in a one-hour period follows a Poisson distribution with mean 3.2.
 - Find $P(X = 5)$.
 - Find $P(X \leq 2)$.
 - State $\text{Var}(X)$. [5]
- Calls to a helpline arrive independently at a mean rate of 2.1 calls per 10-minute period, and can be modelled by a Poisson distribution.
Find the probability that at least one call is received in a randomly chosen 10-minute period. [3]
- Let $X \sim \text{Po}(3.2)$ and $Y \sim \text{Po}(2.1)$ be independent random variables, representing the number of emails and calls received in an hour respectively. Let $T = X + Y$.
 - State the distribution of T , including its parameter.
 - Find $P(T = 4)$. [4]
- The number of flaws, W , in a metre of fabric follows a Poisson distribution with mean m . Given that $P(W = 0) = 0.0498$, find the value of m . [3]

Section B – Confidence Intervals and Hypothesis Testing (Calculator (GDC) required)

- A random sample of 40 students sat a test. The sample mean score was 68 and the sample standard deviation was 12.
Assuming the sample size is large enough to use a normal approximation, construct a 95% confidence interval for the population mean score, giving the endpoints correct to one decimal place. [4]
- A random sample of 15 observations from a normal population has mean 52 and standard deviation 6. It is claimed the population mean is 48. Test, at the 5% significance level, whether the sample provides evidence that the population mean exceeds 48.
State your hypotheses, calculate the test statistic, and state your conclusion (critical value $t_{14} = 1.761$ at the 5% level, one-tailed). [5]
- Explain what is meant by:
 - a Type I error,
 - a Type II error,in the context of a hypothesis test. [2]
- A fair six-sided die is rolled 60 times, so the expected frequency for each face is 10. The observed frequencies for faces 1 to 6 are 4, 12, 9, 15, 11, 9.
Carry out a χ^2 goodness-of-fit test at the 5% significance level to determine whether the die is fair. The critical value is $\chi^2_5 = 11.07$. [5]

9. A researcher wants to estimate a population mean, given that the population standard deviation is known to be 8. Find the minimum sample size required so that a 95% confidence interval for the mean has a margin of error of at most 2. [4]

Section C – Transition Matrices (Calculator (GDC) required)

10. In a simplified weather model, if a day is sunny the probability the next day is sunny is 0.7 (otherwise rainy); if a day is rainy the probability the next day is sunny is 0.4 (otherwise rainy). Today is sunny.
- (a) Write down the transition matrix T for this Markov chain, using the state vector $\begin{pmatrix} P(\text{sunny}) \\ P(\text{rainy}) \end{pmatrix}$. [5]
- (b) Find the probability that it is sunny two days from today.
11. Using the weather model above, find the long-run (steady-state) probability that a day is sunny. [4]
12. Explain, in the context of the weather model, what is meant by a steady-state (equilibrium) probability vector, and state one condition on the transition matrix needed for a unique steady-state to exist. [3]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

- $P(X = 5) = 0.114$.
 - $P(X \leq 2) = 0.380$.
 - $\text{Var}(X) = 3.2$ (mean = variance for a Poisson distribution).
- $P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-2.1} = 0.878$.
- $T \sim \text{Po}(5.3)$.
 - $P(T = 4) = 0.164$.
- $e^{-m} = 0.0498 \Rightarrow m = -\ln(0.0498) = 3.00$.
- $68 \pm 1.96 \times \frac{12}{\sqrt{40}} = 68 \pm 3.72$, giving the interval (64.3, 71.7).
- $H_0 : \mu = 48, H_1 : \mu > 48$. Test statistic $t = \frac{52 - 48}{6/\sqrt{15}} = 2.58$. Since $2.58 > 1.761$, reject H_0 : there is significant evidence at the 5% level that the population mean exceeds 48.
- A Type I error is rejecting H_0 when H_0 is actually true.
 - A Type II error is failing to reject H_0 when H_0 is actually false.
- H_0 : the die is fair; H_1 : the die is not fair. $\chi^2_{\text{calc}} = \sum \frac{(O - E)^2}{E} = 6.80$. Since $6.80 < 11.07$, do not reject H_0 : there is insufficient evidence that the die is biased.
- $n \geq \left(\frac{1.96 \times 8}{2} \right)^2 = 61.5$, so the minimum sample size is $n = 62$.
- $T = \begin{pmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{pmatrix}$ acting on the state column vector.
 - $P(\text{sunny in 2 days}) = 0.61$.
- Solving $\pi_1 = 0.7\pi_1 + 0.4\pi_2$ with $\pi_1 + \pi_2 = 1$ gives $\pi_1 = \frac{4}{7} = 0.571$ and $\pi_2 = \frac{3}{7} = 0.429$. Long-run $P(\text{sunny}) = \frac{4}{7} \approx 0.571$.
- A steady-state vector π satisfies $\pi T = \pi$: once reached, the probabilities of being in each state no longer change from one step to the next, regardless of the initial state. A unique steady state exists provided the transition matrix is regular (some power of T has all entries strictly positive).

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