

# IB Maths AI SL – Topic 1: Number and Algebra

IB AI SL · Total marks: 66 · Answers on the last page

Covers approximation and percentage error, arithmetic and geometric sequences, financial mathematics using the TVM solver, exponents and logarithms. A GDC is allowed throughout; give answers to 3 significant figures unless stated. Allow about 70 minutes.

## Section A – Skills (GDC allowed. Short answers.)

1. (a) Write 0.0034567 correct to 3 significant figures.  
(b) Write your answer to (a) in the form  $a \times 10^k$  where  $1 \leq a < 10$  and  $k \in \mathbb{Z}$ .  
(c) A student measures a mass as 12.5 g. The exact mass is 12.32 g. Calculate the percentage error in the student's measurement. [5]
2. An arithmetic sequence has first term 7 and common difference 4.  
(a) Find the 20th term.  
(b) Find the sum of the first 20 terms.  
(c) Find the smallest value of  $n$  for which  $u_n > 200$ . [6]
3. The first three terms of a geometric sequence are 3, 6, 12, ...  
(a) Write down the common ratio.  
(b) Find the 10th term.  
(c) Find the sum of the first 10 terms. [5]
4. Evaluate  $\sum_{k=1}^{15} (3k + 2)$ . [3]
5. (a) Solve  $2^{x+1} = 50$ , giving your answer to 3 significant figures.  
(b) Without a calculator, simplify  $\log_{10} 8 + \log_{10} 5 - \log_{10} 4$ .  
(c) Write  $\frac{x^3 \sqrt{x}}{x-2}$  in the form  $x^n$ . [6]

## Section B – Exam-style (GDC allowed. Show your method, including any TVM parameters used.)

6. Maya invests 5000 USD in an account paying 4.2% per annum, compounded quarterly.  
(a) Find the value of the investment after 6 years.  
(b) Find the total interest earned.  
(c) Find how many years it takes for the investment to double in value (compounded quarterly). [7]
7. Ben takes out a loan of 15000 EUR at 6% per annum, compounded monthly, to be repaid in equal monthly payments over 5 years.  
(a) Find the monthly payment.  
(b) Find the total amount of interest Ben pays over the 5 years. [5]
8. A car is bought for 24000 GBP and depreciates by 15% each year.  
(a) Write down an expression for the value  $V$  of the car after  $n$  years.  
(b) Find the value of the car after 4 years.  
(c) Find the number of complete years after which the value first falls below 10000 GBP. [6]
9. Aisha saves 200 AUD at the end of every month into an account paying 3% per annum, compounded monthly.

- (a) Find the value of her savings after 10 years.
- (b) State the total amount Aisha deposited and hence find the interest earned. [5]

10. A savings account pays a nominal rate of 4.5% per annum. Inflation is 2.1% per annum.

- (a) Explain what is meant by the real rate of return.
- (b) Calculate the real rate of return, giving your answer to 2 decimal places. [4]

### Section C – Challenge (Extended response.)

11. The side of a square tile is measured as 12 cm, correct to the nearest cm.

- (a) Write down the lower and upper bounds of the side length.
- (b) Find the lower and upper bounds of the area of the tile.
- (c) A rectangular floor is measured as 8.3 m by 5.1 m, each correct to 2 significant figures. The exact dimensions are 8.34 m by 5.08 m. Calculate the percentage error in the area found using the rounded measurements. [7]

12. A population of bacteria grows so that its size after  $t$  hours is  $N = 500 \times 1.8^t$ .

- (a) Find the population after 5 hours.
- (b) Find the time, in hours, for the population to reach 100000. Give your answer to 3 significant figures.
- (c) A second colony has population  $M = 20000 \times 0.7^t$ . Use logarithms or your GDC to find the time when the two populations are equal. [7]

## Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

- 0.00346
  - $3.46 \times 10^{-3}$
  - $\left| \frac{12.5-12.32}{12.32} \right| \times 100 = 1.46\%$
- $u_{20} = 7 + 19(4) = 83$
  - $S_{20} = \frac{20}{2}(14 + 76) = 900$
  - $7 + 4(n - 1) > 200 \Rightarrow n > 49.25$ , so  $n = 50$
- $r = 2$
  - $u_{10} = 3 \times 2^9 = 1536$
  - $S_{10} = \frac{3(2^{10}-1)}{2-1} = 3069$
- $3 \times 120 + 2 \times 15 = 390$  (arithmetic series,  $u_1 = 5$ ,  $u_{15} = 47$ ,  $S_{15} = \frac{15}{2}(5 + 47) = 390$ )
- $x + 1 = \log_2 50 \Rightarrow x = 4.64$
  - $\log_{10} \frac{8 \times 5}{4} = \log_{10} 10 = 1$
  - $x^{3+0.5+2} = x^{5.5}$
- $5000 \left(1 + \frac{0.042}{4}\right)^{24} = 6424.53$  USD (accept 6420)
  - 1424.53 USD
  - $N$  from TVM:  $n = 66.4$  quarters, so about 16.6 years (accept 17 years)
- TVM:  $N = 60$ ,  $I = 6$ ,  $PV = 15000$ ,  $FV = 0$ ,  $P/Y = C/Y = 12 \Rightarrow PMT = 289.99$  EUR (accept 290)
  - $60 \times 289.99 - 15000 = 2399.52$  EUR (accept 2400)
- $V = 24000(0.85)^n$
  - 12528 GBP (accept 12500)
  - $0.85^n < \frac{10000}{24000} \Rightarrow n > 5.39$ , so 6 years
- TVM:  $N = 120$ ,  $I = 3$ ,  $PV = 0$ ,  $PMT = -200$ ,  $P/Y = C/Y = 12 \Rightarrow FV = 27948.28$  AUD (accept 27900)
  - Deposited 24000 AUD; interest = 3948.28 AUD (accept 3950)
- The rate of growth of the purchasing power of the money, i.e. the interest rate after adjusting for inflation.
  - $\frac{1.045}{1.021} - 1 = 2.35\%$  (accept the approximation  $4.5 - 2.1 = 2.4\%$ )
- $11.5 \leq L < 12.5$  cm
  - $132.25 \leq A < 156.25$  cm<sup>2</sup>
  - Rounded area 42.33; exact 42.3672; percentage error = 0.0878%
- $500 \times 1.8^5 = 9447.84 \approx 9450$
  - $1.8^t = 200 \Rightarrow t = \frac{\ln 200}{\ln 1.8} = 9.01$  hours
  - $500 \times 1.8^t = 20000 \times 0.7^t \Rightarrow \left(\frac{1.8}{0.7}\right)^t = 40 \Rightarrow t = \frac{\ln 40}{\ln(18/7)} = 3.91$  hours

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