

IB Maths AI SL – Topic 2: Functions

IB AI SL · Total marks: 72 · Answers on the last page

Covers linear, quadratic, exponential and sinusoidal models, inverse and composite functions, and using a GDC to solve model equations. GDC allowed throughout; allow about 75 minutes. Give answers to 3 significant figures unless exact.

Section A – Skills (Short questions. GDC allowed.)

1. The line L passes through $A(2, 5)$ and $B(6, 13)$.
 - (a) Find the gradient of L .
 - (b) Write the equation of L in the form $y = mx + c$.
 - (c) Find the x -intercept of L . [5]
2. Let $f(x) = 3x - 4$.
 - (a) Find $f^{-1}(x)$.
 - (b) Find $f^{-1}(11)$.
 - (c) State the domain of f^{-1} . [4]
3. Let $f(x) = x^2 + 1$ and $g(x) = 2x - 3$.
 - (a) Find $f(g(2))$.
 - (b) Find $(g \circ f)(x)$ in simplified form.
 - (c) Solve $(g \circ f)(x) = 15$. [5]
4. The function $f(x) = -2x^2 + 120x - 1000$ models the daily profit, in dollars, of a stall selling x items.
 - (a) Write down the equation of the axis of symmetry.
 - (b) Find the maximum daily profit.
 - (c) Find the values of x for which the profit is zero. [6]
5. A population of bacteria is modelled by $N(t) = 5000 \times 1.06^t$, where t is the time in hours.
 - (a) Write down the initial population.
 - (b) State the percentage growth rate per hour.
 - (c) Find the time taken for the population to reach 8000. [5]

Section B – Modelling (Show your method. GDC allowed.)

6. A taxi company charges a fixed fee plus a rate per kilometre. A journey of 50 km costs \$95 and a journey of 120 km costs \$165.
 - (a) Find a linear model $C = mk + c$ for the cost C of a journey of k km.
 - (b) Interpret the value of c in context.
 - (c) Find the cost of a 200 km journey. [6]
7. A ball is thrown so that its height above the ground, in metres, after t seconds is $h(t) = -5t^2 + 20t + 1.5$.
 - (a) Write down the height from which the ball is thrown.
 - (b) Find the maximum height reached and the time at which it occurs.
 - (c) Find the time at which the ball hits the ground. [6]
8. A cup of coffee cools according to $T(t) = 20 + 60e^{-0.05t}$, where T is the temperature in $^{\circ}\text{C}$ and t is the time in minutes.

- (a) Write down the initial temperature of the coffee.
(b) Find the temperature after 10 minutes.
(c) Find the time taken for the coffee to cool to 30°C .
(d) State the room temperature implied by the model and explain your reasoning. [7]
9. The amount of a drug in the bloodstream, in mg, t hours after a dose is given by $D(t) = 200 \times 0.8^t$.
(a) Find the amount remaining after 6 hours.
(b) Find the half-life of the drug in the bloodstream.
(c) A second dose must be given once the amount falls below 40 mg. Find, to the nearest 0.1 hour, when this happens. [6]
10. The depth of water d metres in a harbour t hours after midnight is modelled by $d(t) = 3 \sin(30t)^{\circ} + 7$.
(a) Write down the maximum and minimum depths.
(b) Find the period of the model.
(c) Find the depth at 02:00.
(d) Find the first time after midnight at which the depth is 10 m. [7]

Section C – Extended (Longer modelling problems.)

11. A Ferris wheel has diameter 20 m and its lowest point is 2 m above the ground. It rotates once every 60 seconds. A passenger boards at the lowest point at $t = 0$.
(a) Write down the height of the centre of the wheel above the ground.
(b) Explain why the height h of the passenger after t seconds can be modelled by $h(t) = 12 - 10 \cos(6t)^{\circ}$.
(c) Find the height of the passenger after 20 seconds.
(d) Find the length of time during one rotation for which the passenger is more than 17 m above the ground. [8]
12. A scientist records the mass m grams of a substance at time t days: $(0, 80)$, $(2, 51.2)$, $(4, 32.8)$, $(6, 21.0)$.
(a) Show that an exponential model $m = 80 \times r^t$ with $r = 0.8$ fits the data at $t = 2$.
(b) Using this model, find the predicted mass at $t = 10$.
(c) Use technology to find the exponential regression model $m = ab^t$ for the data and state a and b to 3 significant figures.
(d) State one limitation of using this model to predict the mass after 50 days. [7]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

1. (a) $m = 2$
(b) $y = 2x + 1$
(c) $x = -0.5$
2. (a) $f^{-1}(x) = \frac{x+4}{3}$
(b) 5
(c) $x \in \mathbb{R}$
3. (a) $f(1) = 2$
(b) $2x^2 - 1$
(c) $x = \pm\sqrt{8} = \pm 2.83$
4. (a) $x = 30$
(b) \$800
(c) $x = 10$ and $x = 50$
5. (a) 5000
(b) 6 percent per hour
(c) $t = \frac{\ln 1.6}{\ln 1.06} = 8.07$ hours
6. (a) $m = 1$, $c = 45$, so $C = k + 45$
(b) The fixed fee of \$45 charged regardless of distance
(c) \$245
7. (a) 1.5 m
(b) $t = 2$ s, $h = 21.5$ m
(c) $t = 4.07$ s (positive root of $-5t^2 + 20t + 1.5 = 0$)
8. (a) 80°C
(b) $20 + 60e^{-0.5} = 56.4^\circ\text{C}$
(c) $e^{-0.05t} = \frac{1}{6}$, $t = 35.8$ minutes
(d) 20°C ; as $t \rightarrow \infty$, $e^{-0.05t} \rightarrow 0$ so $T \rightarrow 20$ (horizontal asymptote)
9. (a) $200(0.8)^6 = 52.4$ mg
(b) $\frac{\ln 0.5}{\ln 0.8} = 3.11$ hours
(c) $0.8^t = 0.2$, $t = 7.2$ hours
10. (a) Maximum 10 m, minimum 4 m
(b) 12 hours
(c) $3 \sin 60^\circ + 7 = 9.60$ m
(d) $\sin(30t)^\circ = 1$, so $t = 3$ (03:00)
11. (a) 12 m
(b) Amplitude = 10 (radius), midline = 12, period = 60 s gives $\frac{360}{60} = 6$; negative cosine because start at minimum
(c) $12 - 10 \cos 120^\circ = 17$ m
(d) $h > 17$ when $\cos(6t)^\circ < -0.5$, i.e. $120^\circ < 6t < 240^\circ$, so $20 < t < 40$: 20 seconds
12. (a) $80 \times 0.8^2 = 51.2$ (correct)
(b) $80 \times 0.8^{10} = 8.59$ g

(c) $a \approx 80.0$, $b \approx 0.800$

(d) Extrapolation far beyond the data; model never reaches zero / measurement limits

Need this topic taught, not just practised? Book a trial lesson at 50% off with Mr. Ahmad — online worldwide or in person in Dubai.

WhatsApp **+971 55 373 5695** · mathandsciencetutor.com