

IB Maths AI SL – Topic 5: Calculus

IB Mathematics: Applications and Interpretation SL · Total marks: 50 · Answers on the last page

Practice for Topic 5 (differentiation, integration, the trapezoidal rule and optimisation) in the style of Paper 1 and Paper 2. A GDC is allowed throughout; give answers to 3 significant figures unless exact. Allow about 75 minutes.

Section A – Skills (Short questions on differentiation and integration.)

- Find $\frac{dy}{dx}$ when $y = 3x^4 - 2x^2 + 5x - 7$. [2]
- The function f is defined by $f(x) = 2x^3 + \frac{1}{x}$, $x \neq 0$.
 - Find $f'(x)$.
 - Find the gradient of the curve at $x = 2$. [3]
- The curve $y = x^2 - 4x$ passes through the point $P(3, -3)$.
 - Find the gradient of the curve at P .
 - Hence find the equation of the tangent to the curve at P in the form $y = mx + c$. [3]
- Find $\int (6x^2 - 4x + 3) dx$. [2]
- Evaluate $\int_1^3 (2x + 1) dx$. [3]
- A function has derivative $f'(x) = 4x - 3$ and passes through the point $(2, 5)$. Find $f(x)$. [3]

Section B – Exam-style

- Consider the curve $y = x^3 - 6x^2 + 9x + 1$.
 - Find $\frac{dy}{dx}$.
 - Find the coordinates of the two stationary points of the curve.
 - Determine the nature of each stationary point, justifying your answer. [6]
- Let $f(x) = \sqrt{x^2 + 1}$.
 - Complete a table of values of $f(x)$ for $x = 0, 0.5, 1, 1.5, 2$, giving values to 4 decimal places.
 - Use the trapezoidal rule with four intervals to estimate $\int_0^2 \sqrt{x^2 + 1} dx$.
 - Use your GDC to find the exact integral to 4 significant figures, and hence find the percentage error in your estimate. [6]
- A company sells x units of a product per week. The weekly revenue is $R(x) = 80x - 0.5x^2$ dollars and the weekly cost is $C(x) = 20x + 500$ dollars.
 - Write down an expression for the weekly profit $P(x)$.
 - Find $\frac{dP}{dx}$.
 - Find the number of units that maximises the weekly profit, and the maximum profit. [5]

10. The curve $y = 4x - x^2$ meets the x -axis at the origin and at the point A .
- (a) Find the x -coordinate of A .
 - (b) Find the area of the region enclosed by the curve and the x -axis. [4]

Section C – Challenge

11. A cylindrical can with a closed top and bottom is to hold 500 cm^3 of drink. The radius of the can is r cm and its height is h cm.
- (a) Show that $h = \frac{500}{\pi r^2}$.
 - (b) Show that the total surface area of the can is $S = 2\pi r^2 + \frac{1000}{r}$.
 - (c) Find $\frac{dS}{dr}$.
 - (d) Find the value of r that minimises the surface area, and the corresponding minimum surface area. [7]
12. A surveyor measures the depth of a river at 2 m intervals across its width. The depths (in metres) are 0, 1.2, 2.5, 3.1, 2.8, 1.6, 0.
- (a) Use the trapezoidal rule to estimate the cross-sectional area of the river.
 - (b) The water flows at an average speed of 0.5 m s^{-1} . Estimate the volume of water passing a point on the river each second.
 - (c) Explain how the surveyor could obtain a more accurate estimate of the area. [6]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

- $\frac{dy}{dx} = 12x^3 - 4x + 5$
- (a) $f'(x) = 6x^2 - \frac{1}{x^2}$
(b) $f'(2) = 24 - 0.25 = 23.75$
- (a) $\frac{dy}{dx} = 2x - 4$, gradient = 2
(b) $y = 2x - 9$
- $2x^3 - 2x^2 + 3x + C$
- $[x^2 + x]_1^3 = 12 - 2 = 10$
- $f(x) = 2x^2 - 3x + 3$ (since $8 - 6 + c = 5$ gives $c = 3$)
- (a) $\frac{dy}{dx} = 3x^2 - 12x + 9 = 3(x - 1)(x - 3)$
(b) $(1, 5)$ and $(3, 1)$
(c) $(1, 5)$ is a local maximum (gradient changes + to -, or $\frac{d^2y}{dx^2} = -6 < 0$); $(3, 1)$ is a local minimum ($\frac{d^2y}{dx^2} = 6 > 0$)
- (a) 1, 1.1180, 1.4142, 1.8028, 2.2361
(b) $\frac{0.5}{2} [1 + 2.2361 + 2(1.1180 + 1.4142 + 1.8028)] = 2.98$ (2.9765)
(c) Exact = 2.958; percentage error $\approx 0.63\%$
- (a) $P(x) = 60x - 0.5x^2 - 500$
(b) $\frac{dP}{dx} = 60 - x$
(c) $x = 60$ units; maximum profit = \$1300
- (a) $x = 4$
(b) $\int_0^4 (4x - x^2) dx = \left[2x^2 - \frac{x^3}{3} \right]_0^4 = \frac{32}{3} = 10.7$
- (a) $V = \pi r^2 h = 500 \Rightarrow h = \frac{500}{\pi r^2}$
(b) $S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + \frac{1000}{r}$
(c) $\frac{dS}{dr} = 4\pi r - \frac{1000}{r^2}$
(d) $r^3 = \frac{250}{\pi}$, $r = 4.30$ cm; $S_{\min} = 349$ cm² (348.7); ($h = 8.60$ cm)
- (a) $\frac{2}{2} [0 + 0 + 2(1.2 + 2.5 + 3.1 + 2.8 + 1.6)] = 22.4$ m²
(b) $22.4 \times 0.5 = 11.2$ m³ s⁻¹
(c) Take depth readings at smaller intervals (more strips)

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