

Pre-Calculus – Functions and Trigonometry

Pre-Calculus (US high school / AP preparation) · Total marks: 36 · Answers on the last page

This worksheet covers polynomial, rational, exponential and trigonometric functions. A calculator is allowed except where a question states otherwise, and you should aim to spend about 45 minutes on the full set.

Section A – Functions Skills (Calculator allowed)

1. State the domain of $f(x) = \frac{x+3}{x^2-9}$. [2]
2. Describe the transformation that maps $y = x^2$ to $y = (x-4)^2 + 1$. Hence state the coordinates of the vertex of $y = (x-4)^2 + 1$. [2]
3. Given $f(x) = 2x^3 - 3x^2 - 11x + 6$:
 - (a) Show that $x = 3$ is a root of $f(x) = 0$.
 - (b) Hence factorise $f(x)$ fully. [4]
4. Find the equations of the vertical and horizontal asymptotes of $g(x) = \frac{3x-2}{x+5}$. [3]

Section B – Exponential, Logarithmic and Trigonometric Functions (Calculator allowed)

5. Solve $5^{2x-1} = 125$ algebraically. [3]
6. Solve $\log_2(x+3) + \log_2(x-1) = 5$, stating any restrictions on x . [4]
7. Solve $2\sin\theta - 1 = 0$ for $0 \leq \theta < 360^\circ$. [3]
8. Prove the identity $\frac{1 - \cos^2\theta}{\sin\theta} = \sin\theta$ for $\sin\theta \neq 0$. [2]

Section C – Challenge / Extended (Calculator allowed)

9. A population of bacteria is modelled by $P(t) = 200e^{0.35t}$, where t is measured in hours. Calculate the time taken for the population to reach 1000, giving your answer to 2 decimal places. [4]
10. The height of a ferris wheel car above ground is modelled by $h(t) = 15 - 13\cos\left(\frac{\pi}{6}t\right)$, where h is in metres and t is in minutes. State the maximum height, the minimum height, and the period of rotation. [3]
11. Solve $\tan^2 x - 3 = 0$ for $0 \leq x < \pi$, giving exact answers. [3]
12. A rational function has vertical asymptotes at $x = 2$ and $x = -1$, a horizontal asymptote at $y = 3$, and passes through the origin. Find a possible equation for $f(x)$. [3]

Answers

Check your working against these. If an answer does not match, try the question again before looking for the method.

1. The denominator is zero when $x = 3$ or $x = -3$, so the domain is all real x except $x = 3$ and $x = -3$.
2. Translation of 4 units in the positive x -direction and 1 unit in the positive y -direction. Vertex: $(4, 1)$.
3. (a) $f(3) = 2(27) - 3(9) - 11(3) + 6 = 54 - 27 - 33 + 6 = 0$, so $x = 3$ is a root.
(b) Dividing by $(x - 3)$ gives $2x^2 + 3x - 2 = (2x - 1)(x + 2)$, so $f(x) = (x - 3)(2x - 1)(x + 2)$.
4. Vertical asymptote: $x = -5$ (denominator zero). Horizontal asymptote: $y = 3$ (ratio of leading coefficients, since numerator and denominator have equal degree).
5. $125 = 5^3$, so $2x - 1 = 3$, giving $x = 2$.
6. $\log_2[(x + 3)(x - 1)] = 5 \Rightarrow (x + 3)(x - 1) = 32 \Rightarrow x^2 + 2x - 35 = 0 \Rightarrow (x + 7)(x - 5) = 0$, so $x = -7$ or $x = 5$. Since $x > 1$ is required for both logarithms to be defined, $x = 5$ is the only valid solution.
7. $\sin \theta = 0.5$, so $\theta = 30^\circ$ or $\theta = 150^\circ$.
8. Using $\sin^2 \theta + \cos^2 \theta = 1$, the numerator $1 - \cos^2 \theta = \sin^2 \theta$. So $\frac{\sin^2 \theta}{\sin \theta} = \sin \theta$, as required.
9. $1000 = 200e^{0.35t} \Rightarrow e^{0.35t} = 5 \Rightarrow 0.35t = \ln 5 \Rightarrow t = \frac{\ln 5}{0.35} = 4.60$ hours (2 d.p.).
10. Maximum height = $15 + 13 = 28$ m (when $\cos = -1$). Minimum height = $15 - 13 = 2$ m (when $\cos = 1$). Period = $\frac{2\pi}{\pi/6} = 12$ minutes.
11. $\tan^2 x = 3 \Rightarrow \tan x = \pm\sqrt{3}$. In $0 \leq x < \pi$: $x = \frac{\pi}{3}$ (where $\tan x = \sqrt{3}$) and $x = \frac{2\pi}{3}$ (where $\tan x = -\sqrt{3}$).
12. One possible answer: $f(x) = \frac{3x^2}{(x - 2)(x + 1)}$. This has vertical asymptotes at $x = 2$ and $x = -1$, passes through the origin since $f(0) = 0$, and as $x \rightarrow \pm\infty$, $f(x) \rightarrow 3$ (equal degree numerator and denominator with leading coefficient ratio 3).

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